Mx PRACTICAL: Measurement of complex traits

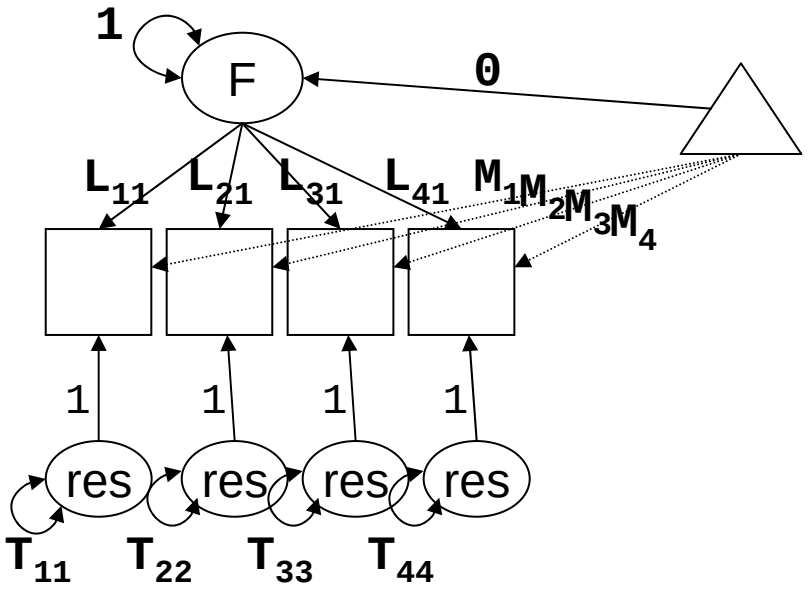
Factor Analysis

&

Measurement Invariance

Phenotypic common factor model

One Group: Full Model



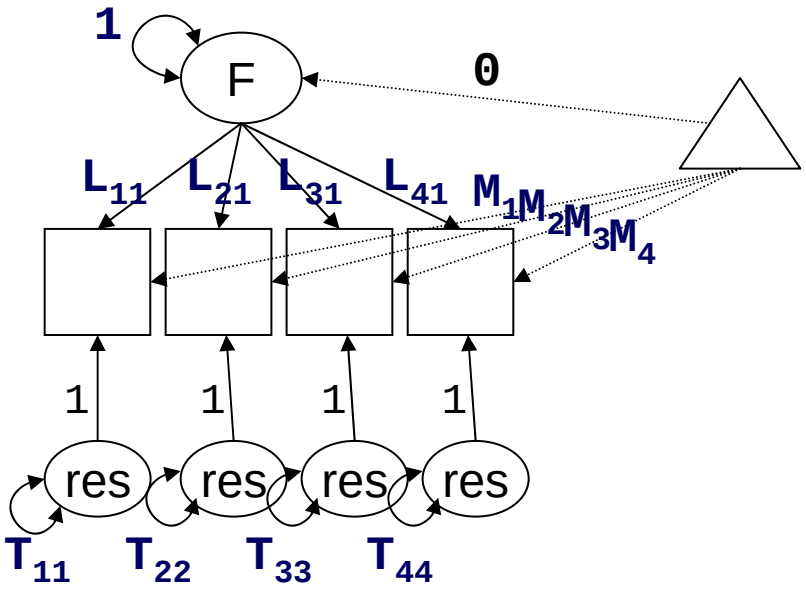
$$\mathbf{L}\begin{bmatrix}1\\ L_{21}\\ L_{31}\\ L_{41}\end{bmatrix}\quad \mathbf{T}\begin{bmatrix}T_{11} & & & \\ & T_{22} & & \\ & & T_{33} & \\ & & & T_{44}\end{bmatrix}\quad \mathbf{P}\ [1]$$

$$\mathbf{M}\ \begin{bmatrix}M_1 & M_2 & M_3 & M_4\end{bmatrix}$$

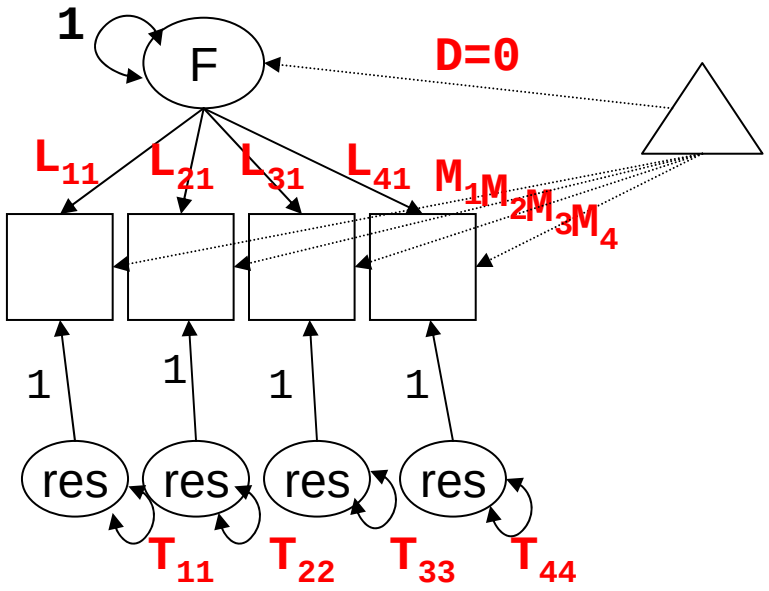
```
Means        M;
Covariance   L*P*L'+T;
```

Phenotypic common factor model

Multigroup: Full Model (M1)



Reference Group: males



Comparison Group: Females

Mx matrices: Full model

Reference Group: males

L $\begin{bmatrix} L_{11} \\ L_{21} \\ L_{31} \\ L_{41} \end{bmatrix}$ **T** $\begin{bmatrix} T_{11} & & & \\ & T_{22} & & \\ & & T_{33} & \\ & & & T_{44} \end{bmatrix}$ **P** $[1]$

M $[M_1 \quad M_2 \quad M_3 \quad M_4]$

```
Means        M;
Covariance   L*P*L'+T;
```

Comparison Group: Females

L $\begin{bmatrix} L_{11} \\ L_{21} \\ L_{31} \\ L_{41} \end{bmatrix}$ **T** $\begin{bmatrix} T_{11} & & & \\ & T_{22} & & \\ & & T_{33} & \\ & & & T_{44} \end{bmatrix}$ **P** $[1]$

D $[d] = [0]$

M $[M_1 \quad M_2 \quad M_3 \quad M_4]$

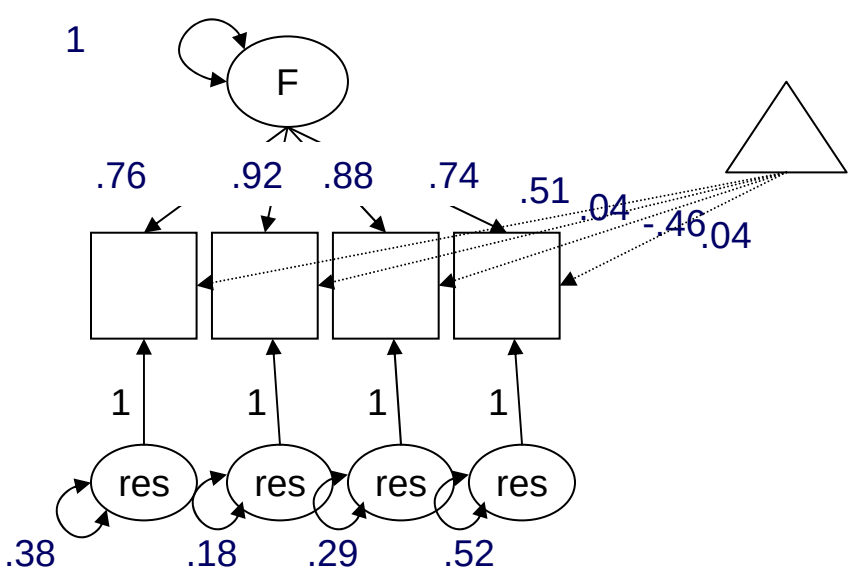
```
Means M+(L*D)';
Covariance  L*P*L'+T;
```

Phenotypic common factor model

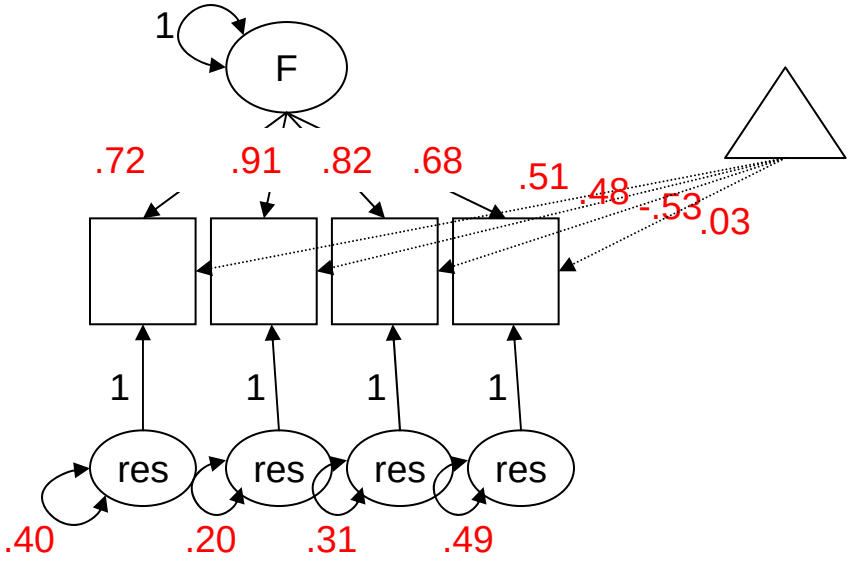
Multigroup: Full Model (M1)

Open: FACTORMI4.mx

Unstandardized estimates Males:



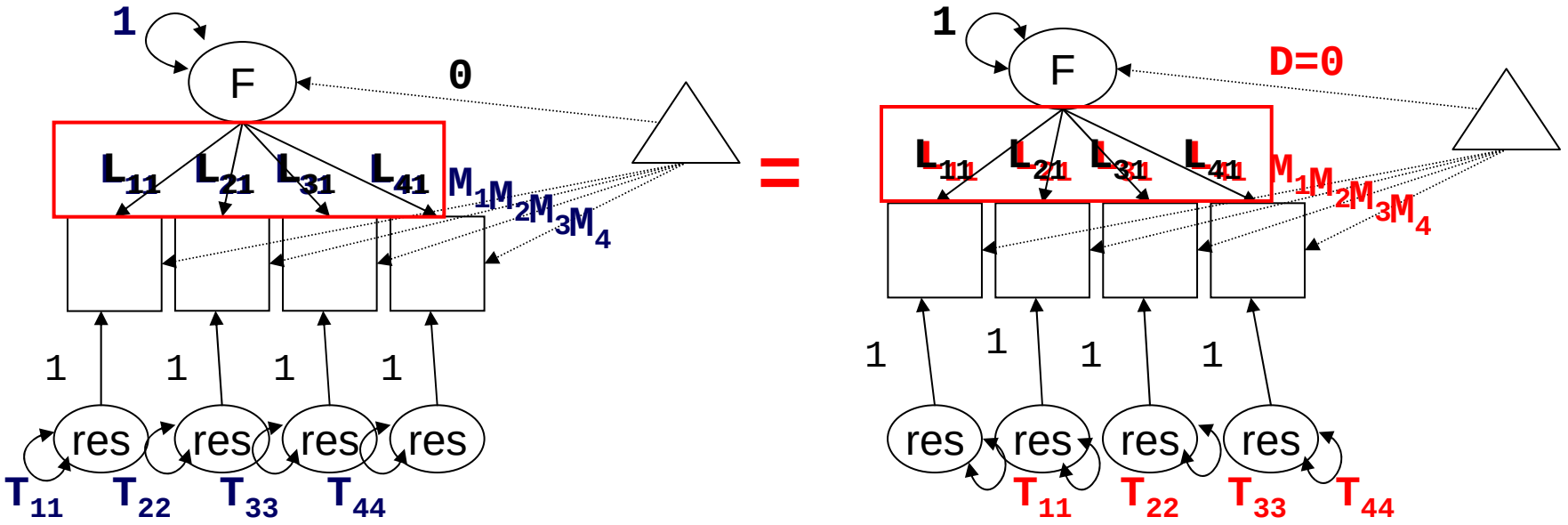
Unstandardized estimates Females:



FIT STATISTICS: -2LL = 9227.349 , DF = 3976 (np=24)

Phenotypic common factor model

Metric Invariance (M2)

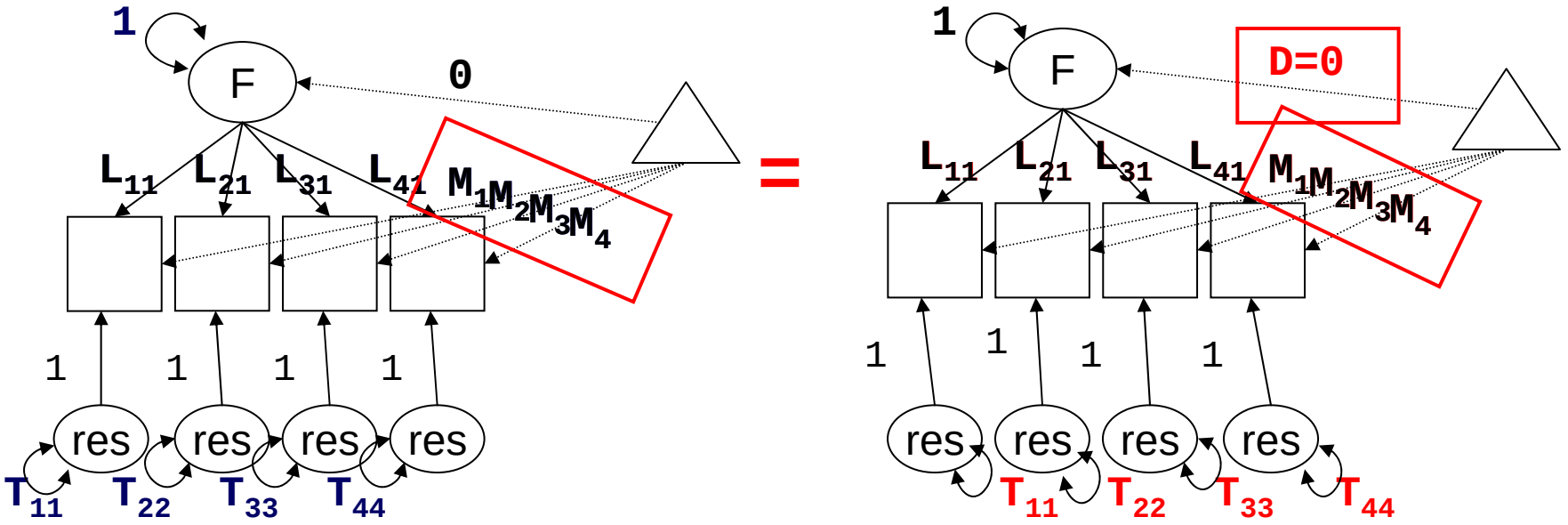


Reference Group: males

Comparison Group: Females

Phenotypic common factor model

Strong Factorial Invariance (M3)

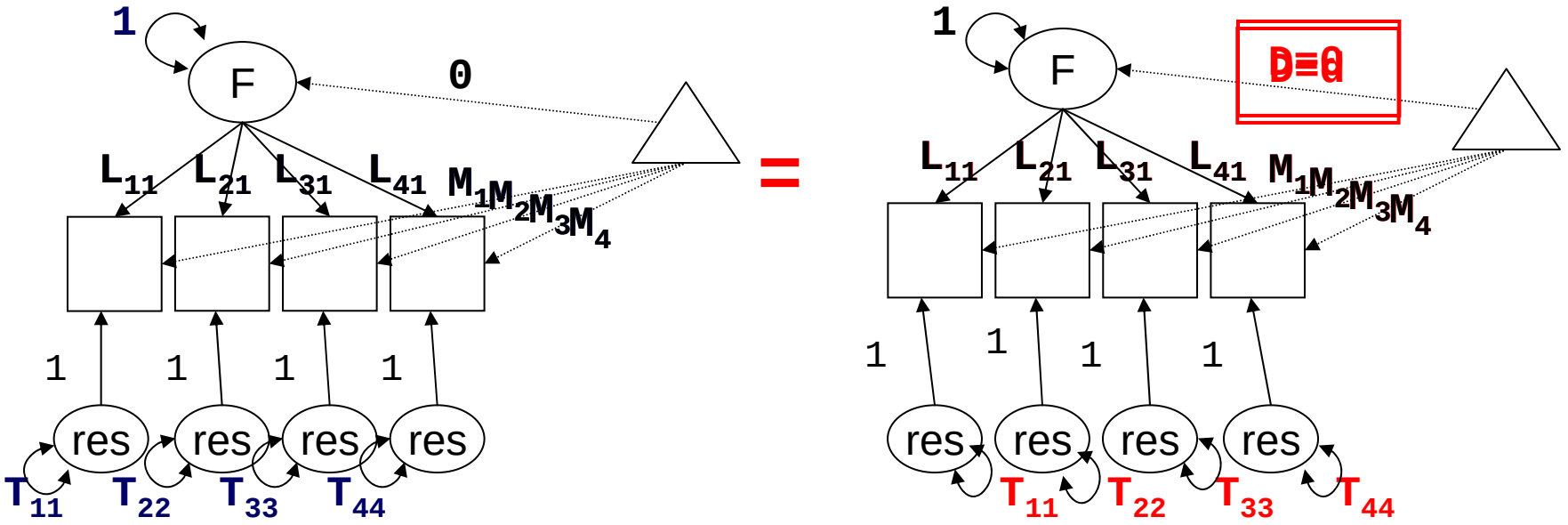


Reference Group: males

Comparison Group: Females

Phenotypic common factor model

Mean differences due to latent facto? (M4)

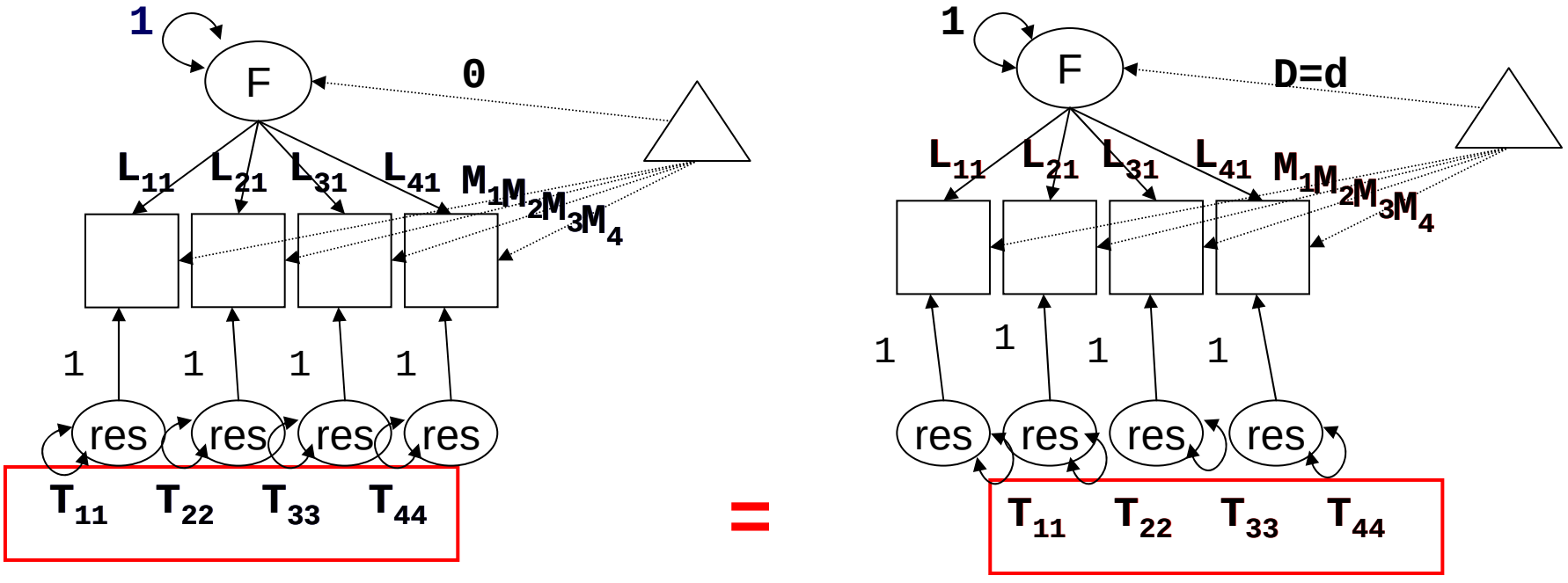


Reference Group: males

Comparison Group: Females

Phenotypic common factor model

Strict Factorial Invariance (M5)



Reference Group: males

Comparison Group: Females

Mx Option Multiple: Testing MI

```
! M2.METRIC INVARIANCE: Constrain factor
loadings accross males and females
eq L 1 1 1 L 2 1 1
eq L 1 2 1 L 2 2 1
eq L 1 3 1 L 2 3 1
eq L 1 4 1 L 2 4 1
End

! M3.STRONG FACTORIAL INVARIANCE: Constrain
the intercepts to be equal for males and
females
eq m 1 1 1 m 2 1 1
eq m 1 1 2 m 2 1 2
eq m 1 1 3 m 2 1 3
eq m 1 1 4 m 2 1 4
end
```

Mx Option Multiple: Testing MI

```
! M4: All difference due to the latent factor?
FREE D 2 1 1
ST 0.5 D 2 1 1
END

! M5: STRICT FACTORIAL INVARIANCE: Constrain
residuals accross males and females
EQ T 1 1 1 T 2 1 1
EQ T 1 2 2 T 2 2 2
EQ T 1 3 3 T 2 3 3
EQ T 1 4 4 T 2 4 4
END
```

Practical: FACTORMI4.mx

	-2LL	DF	CTM	ΔX^2	Δ DF	p
M1.Full Model	9227.349	3976				
M2.Metric Invariance			1			
M3.Strong Invariance			2			
M4. d free			3 (inv)			
M5.Strict Invariance			4			

1. **Can we assume Metric invariance, Strong factorial Invariance or Strict Factorial Invariance?**
2. **Is there a mean difference? Is it due to the latent factor?**

Practical: FACTORMI4.mx

	-2LL	DF	CTM	ΔX^2	Δ DF	p
M1.Full Model	9227.349	3976				
M2.Metric Invariance	9229.296	3980	1	1.947	4	0.746
M3.Strong Invariance	9386.193	3984	2	156.897	4	<.001
M4. d free	9379.239	3983	3 (inv)	6.954	1	0.008
M5.Strict Invariance	9380.706	3987	4	1.467	4	0.832

Practical: Sumscores

Mean & variance differences?

Open: sumscore4.mx

```
G1: Males
Data NInput_vars=6
        REctangular file=sumscore4.dat
        LABELS
  V1 V2 V3 V4 sex sum
  Select if sex = 1 ;
        Select sum;

Begin Matrices;
        M full 1 nvar free              !Mean sumscore
        V full nvar nvar free           !Variance sumscore
End matrices;

st 1 m 1 1
st 10 v 1 1

Means M;
Covariance V;
```

Practical: Sumscores

Mean & variance differences?

```
Option multiple issat
End

!Test variance differences in sumscores
eq v 1 1 1 v 2 1 1
end

!Test mean differences in sumscores
eq m 1 1 1 m 2 1 1
end
```

Practical: Sumscores

Mean & variance differences?

Open: sumscore4.mx → RUN & INTERPRET

	-2LL	DF	CTM	ΔX^2	Δ DF	p
1 Full Model						
2 Equal Variance			1			
3 Equal Mean			2			

Practical: Sumscores

Mean & variance differences?

	-2LL	DF	CTM	ΔX^2	Δ DF	p
1 Full Model	5307.377	996				
2 Equal Variance	5308.298	997	1	0.921	1	0.337
3 Equal Mean	5346.675	998	2	38.377	1	<.001