

# Introduction to QTL mapping

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# Outline

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1. Aim
2. The Human Genome
3. Principles of Linkage Analysis
4. Parametric Linkage Analysis
5. Nonparametric Linkage Analysis

# 1. Aim

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## QTL mapping

- ▷ LOCALIZE and then IDENTIFY a locus that regulates a trait (QTL)



*Nucleotide or sequence of nucleotides with variation in the population,  
with different variants associated with different trait levels.*

For a heritable trait...

**Linkage:** localize region of the genome where a QTL that regulates the trait is likely to be harboured

Family-specific phenomenon:  
Affected individuals in a family share the same ancestral predisposing DNA segment at a given QTL

**Association:** identify a QTL that regulates the trait

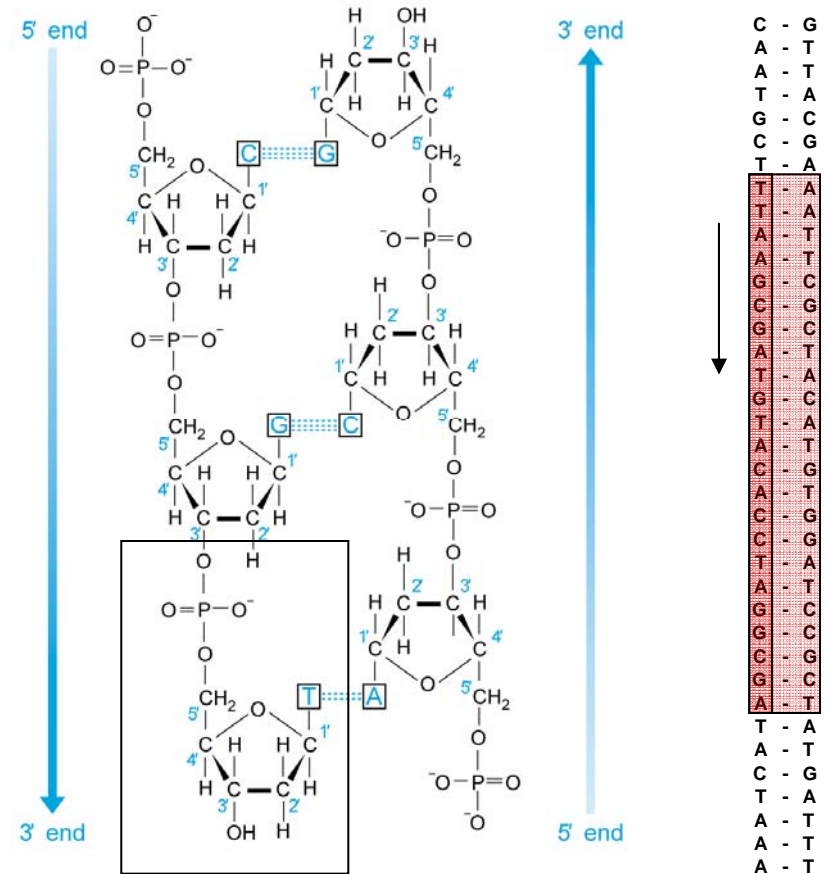
Population-specific phenomenon:  
Affected individuals in a population share the same ancestral predisposing DNA segment at a given QTL

## 2. Human Genome

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# DNA structure

- ▷ A DNA molecule is a linear backbone of alternating sugar residues and phosphate groups
- ▷ Attached to carbon atom 1' of each sugar is a nitrogenous base: A, C, G or T
- ▷ Two DNA molecules are held together in anti-parallel fashion by hydrogen bonds between bases [Watson-Crick rules] Antiparallel double helix
- ▷ A gene is a segment of DNA which is transcribed to give a protein or RNA product
- ▷ Only one strand is read during gene transcription
- ▷ Nucleotide: 1 phosphate group + 1 sugar + 1 base



# DNA polymorphisms

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▷ **Microsatellites**

>100,000

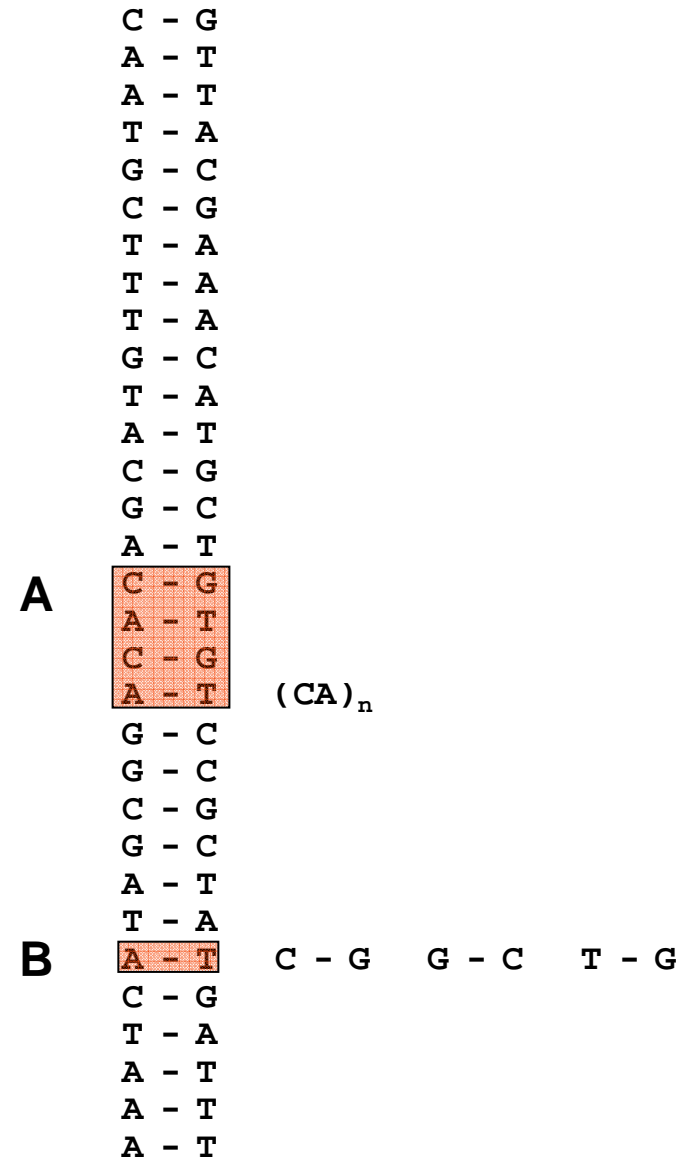
Many alleles,  $(CA)_n$ , very informative, even, easily automated

▷ **SNPs**

10,054,521 (25 Jan '05)

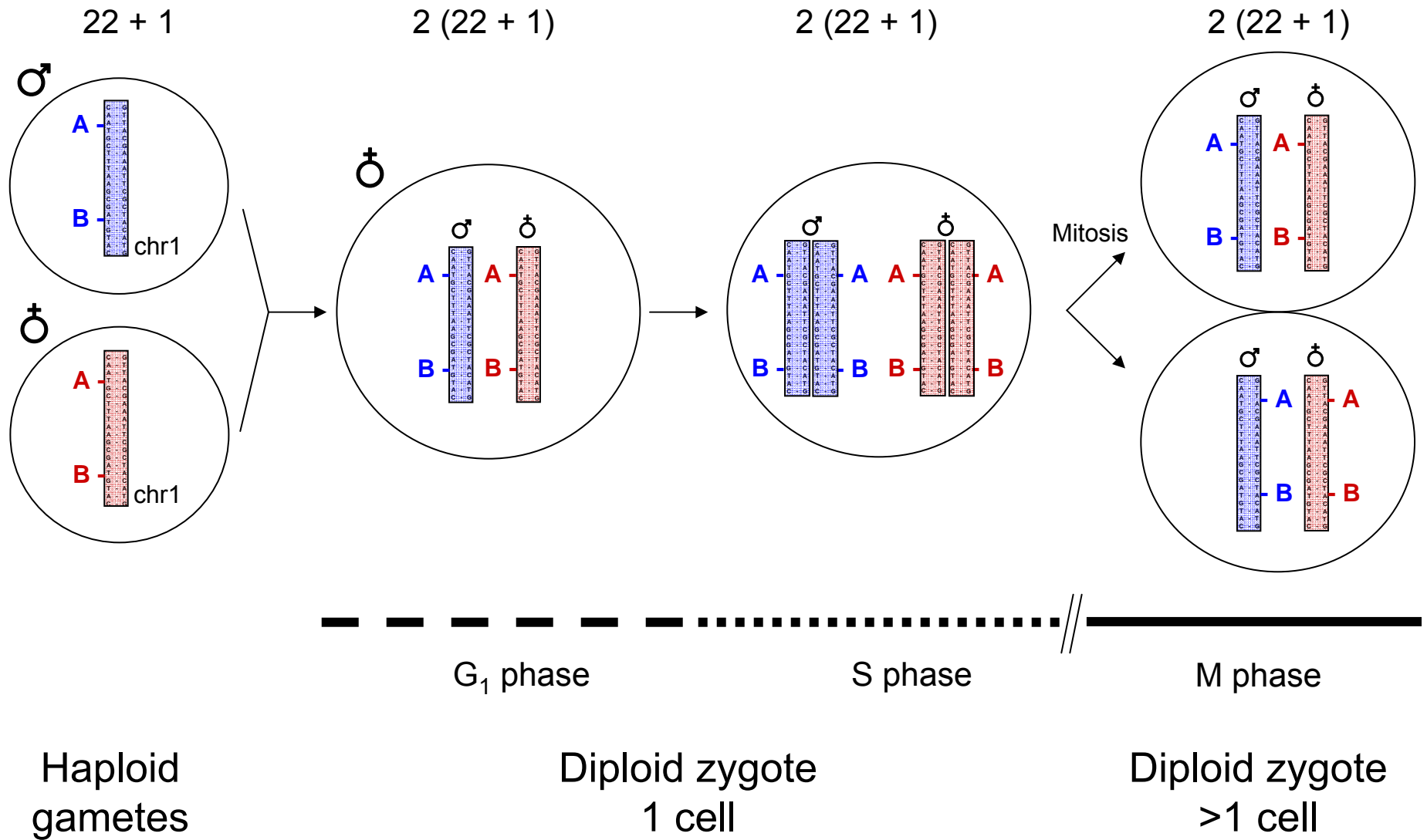
10,430,753 (11 Mar '06)

Most with 2 alleles (up to 4), not very informative, even, easily automated

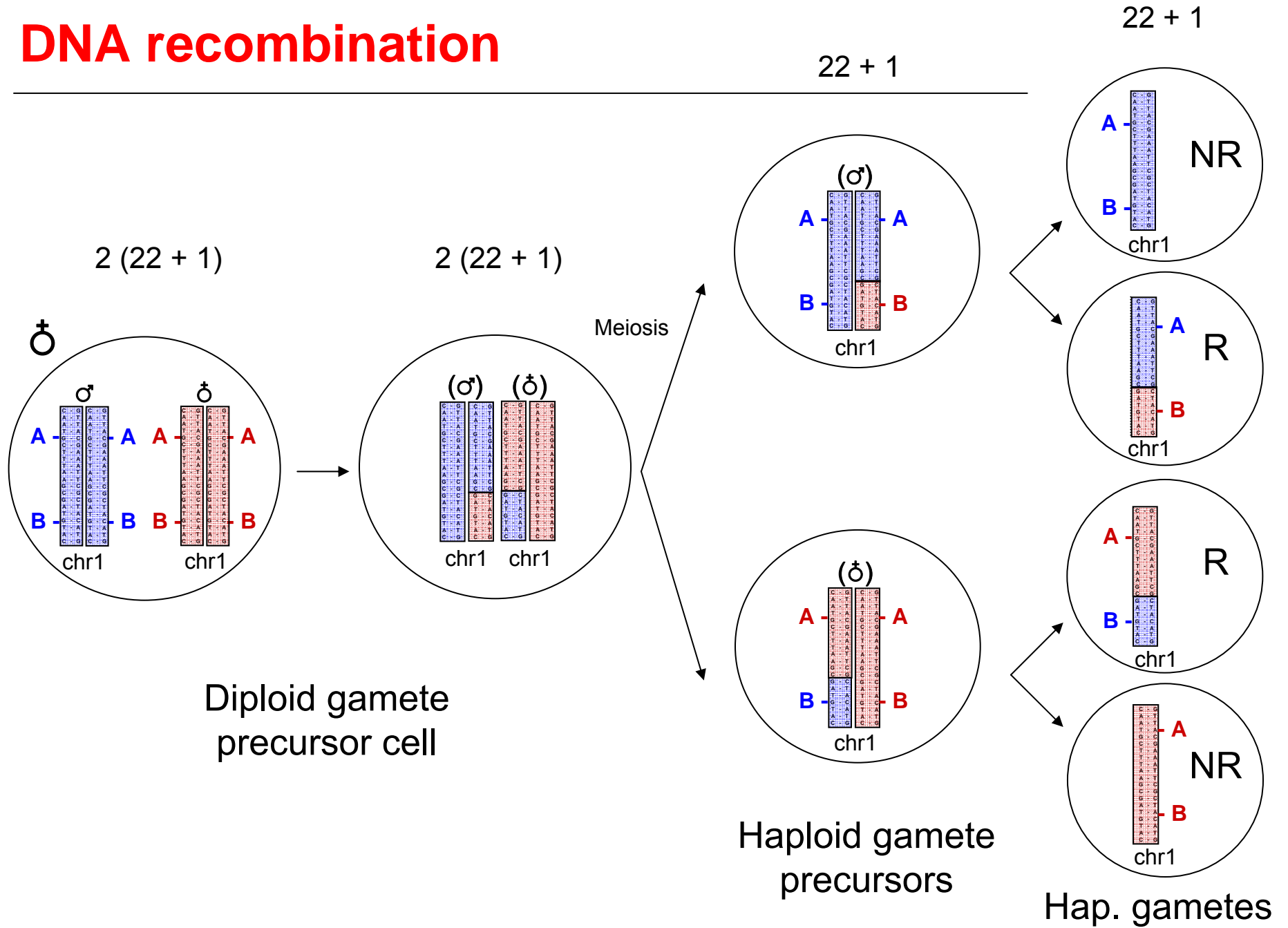




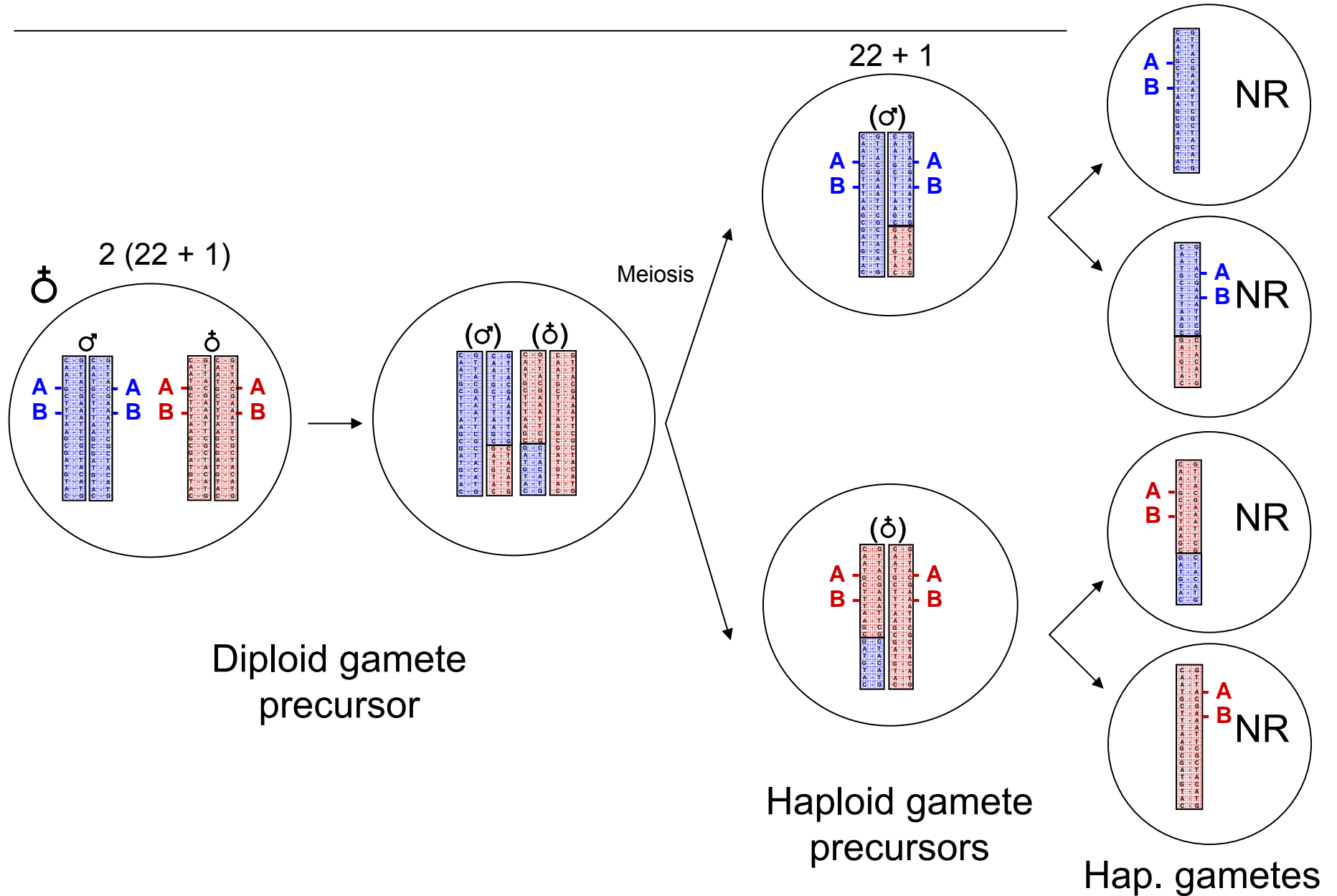
# DNA organization



# DNA recombination



# DNA recombination between linked loci



# Human Genome - summary

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▶ DNA is a linear sequence of nucleotides partitioned into 23 chromosomes

Two copies of each chromosome (2x22 autosomes + XY), from paternal and maternal origins. During meiosis in gamete precursors, recombination can occur between maternal and paternal homologs

▶ Recombination fraction between loci A and B ( $\theta$ )

Proportion of gametes produced that are recombinant for A and B

If A and B are very far apart: 50%R:50%NR -  $\theta = 0.5$

If A and B are very close together: <50%R -  $0 \leq \theta < 0.5$

▶ Recombination fraction ( $\theta$ ) can be converted to genetic distance (cM)

Haldane:  $cM = 100 \cdot [-0.5 \cdot \ln(1 - 2 \cdot \theta)]$  eg.  $\theta=0.17$ ,  $cM=20.8$

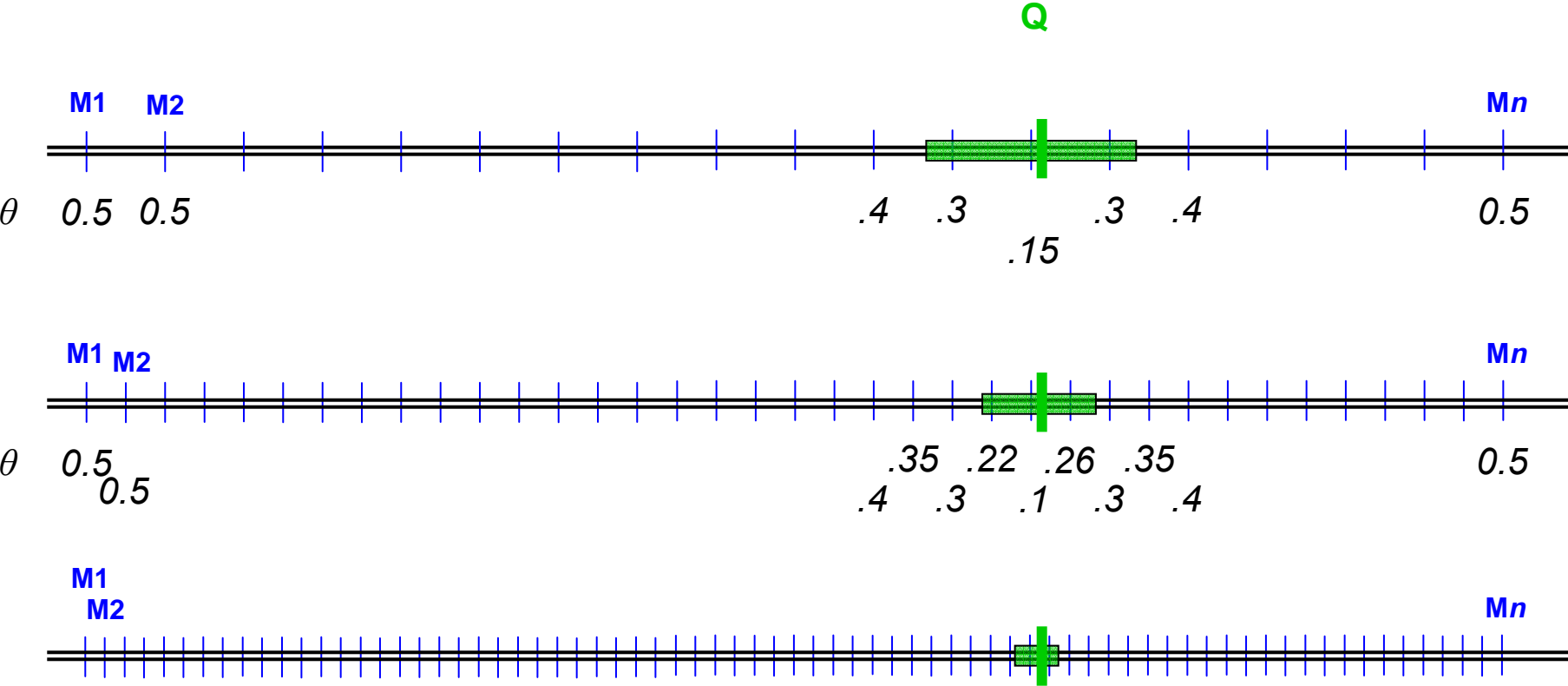
Kosambi:  $cM = 100 \cdot [0.25 \cdot \ln((1 + 2 \cdot \theta)/(1 - 2 \cdot \theta))]$  eg.  $\theta=0.17$ ,  $cM=17.7$

## **3. Principles of Linkage Analysis**

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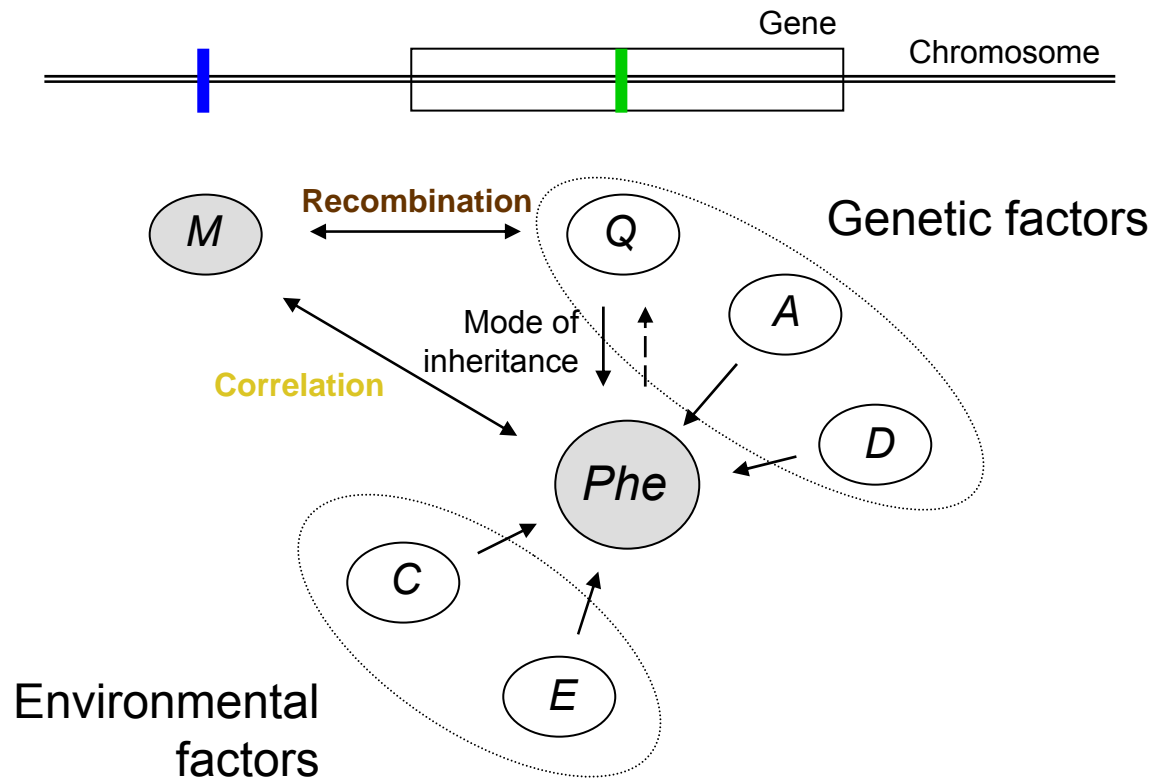
# Linkage Analysis requires genetic markers

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# Linkage Analysis: Parametric vs. Nonparametric

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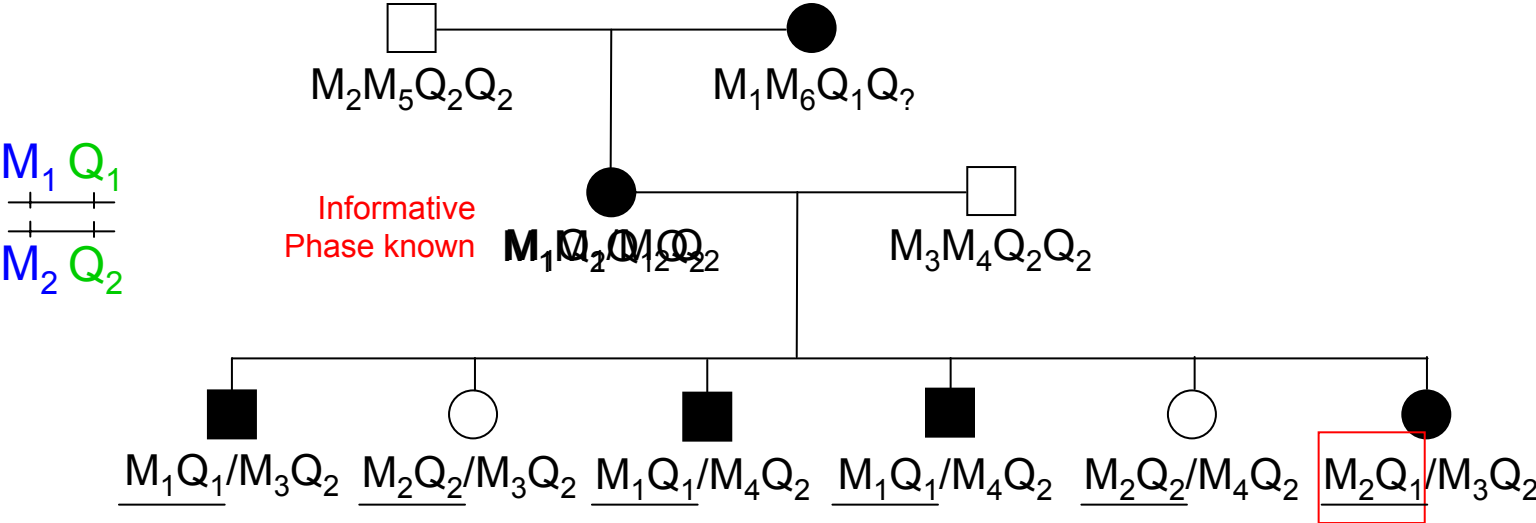
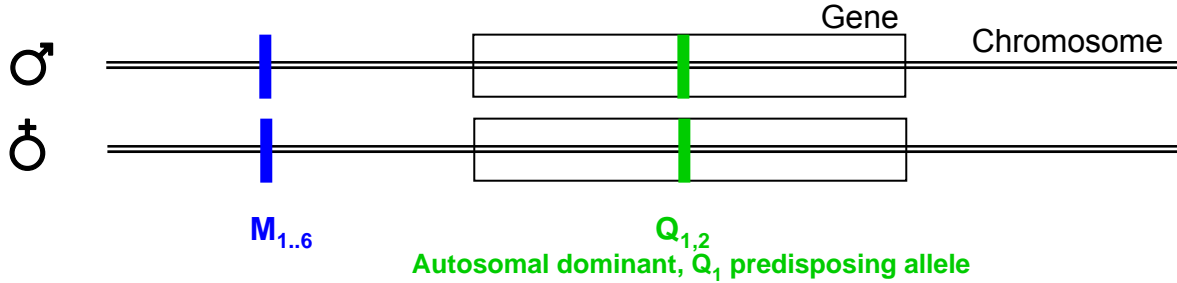


## 4. Parametric Linkage Analysis

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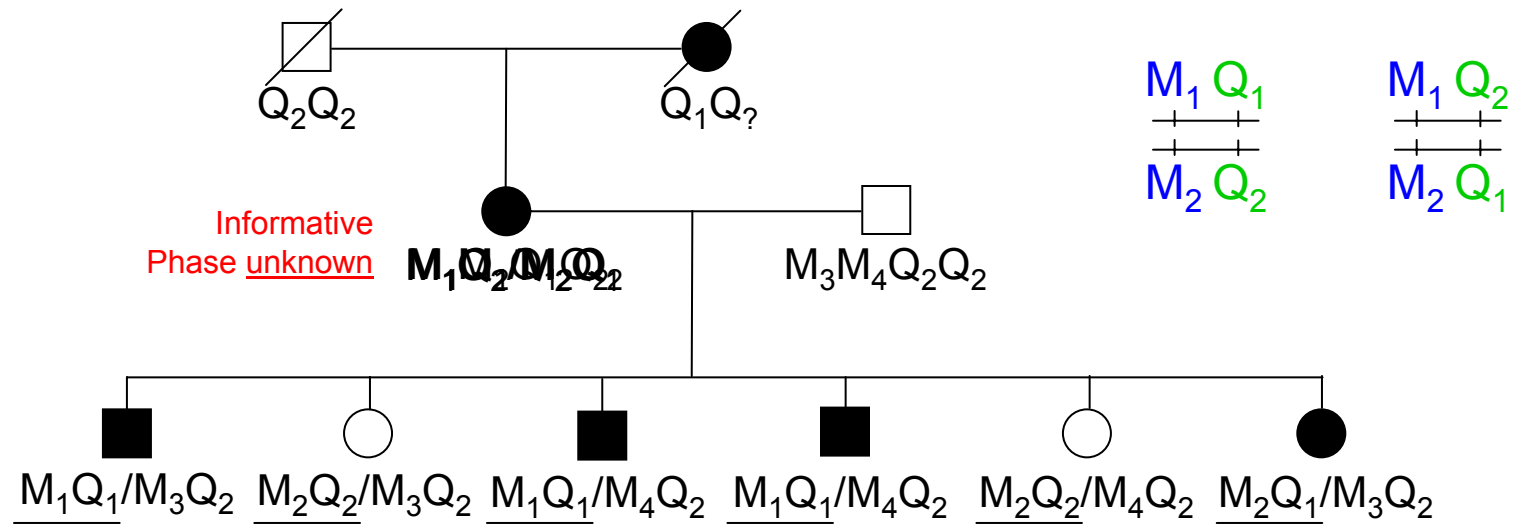
# Linkage with informative phase known meiosis



- NR:  $M_1 Q_1$
- NR:  $M_2 Q_2$
- R:  $M_1 Q_2$
- R:  $M_2 Q_1$

$$\theta_{MQ} = 1/6 = 0.17 \quad (\sim 20.8 \text{ cM})$$

# Linkage with informative phase unknown meiosis



$M_1 Q_1 / M_2 Q_2$	$P$	$N$	$M_1 Q_2 / M_2 Q_1$	$P$	$N$
NR: $M_1 Q_1$	$\frac{1}{2}(1-\theta)$	3	<b>R</b> : $M_1 Q_1$	$\frac{1}{2}\theta$	3
NR: $M_2 Q_2$	$\frac{1}{2}(1-\theta)$	2	<b>R</b> : $M_2 Q_2$	$\frac{1}{2}\theta$	2
<b>R</b> : $M_1 Q_2$	$\frac{1}{2}\theta$	0	NR: $M_1 Q_2$	$\frac{1}{2}(1-\theta)$	0
<b>R</b> : $M_2 Q_1$	$\frac{1}{2}\theta$	1	NR: $M_2 Q_1$	$\frac{1}{2}(1-\theta)$	1

$$L(X | \theta) = \frac{1}{2} \cdot [\theta^1 \cdot (1-\theta)^5] + \frac{1}{2} \cdot [\theta^5 \cdot (1-\theta)^1]$$

$$L(X | \theta = 0.5) = \frac{1}{2} \cdot [0.5^1 \cdot (1-0.5)^5] + \frac{1}{2} \cdot [0.5^5 \cdot (1-0.5)^1] = (0.5)^6$$

# Parametric LOD score calculation

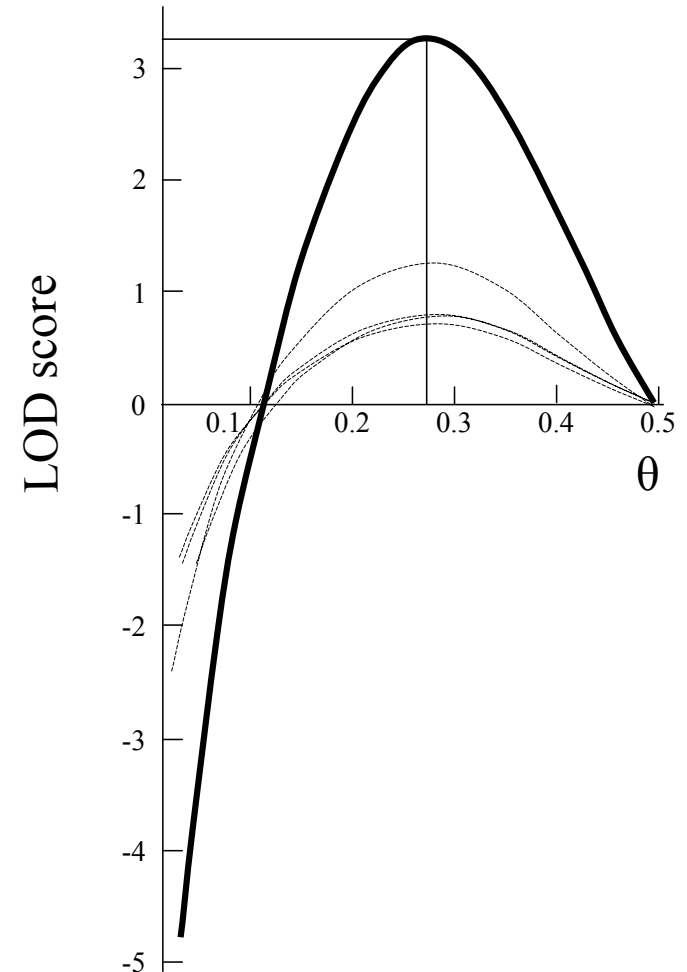
$$OD = \frac{L(X | \theta)}{L(X | \theta = 0.5)} \quad LOD = \log_{10} \frac{L(X | \theta)}{L(X | \theta = 0.5)}$$

$$LOD = \log_{10} \frac{\frac{1}{2} \cdot [\theta^1 \cdot (1-\theta)^5] + \frac{1}{2} \cdot [\theta^5 \cdot (1-\theta)^1]}{(0.5)^6}$$

$$OD = \prod_{i=1}^n \frac{L(X_i | \theta)}{L(X_i | \theta = 0.5)}$$

$$LOD = \log_{10} \left( \prod_{i=1}^n \frac{L(X_i | \theta)}{L(X_i | \theta = 0.5)} \right)$$

$$LOD = \sum_{i=1}^n \log_{10} \left( \frac{L(X_i | \theta)}{L(X_i | \theta = 0.5)} \right) = \sum_{i=1}^n LOD_i$$

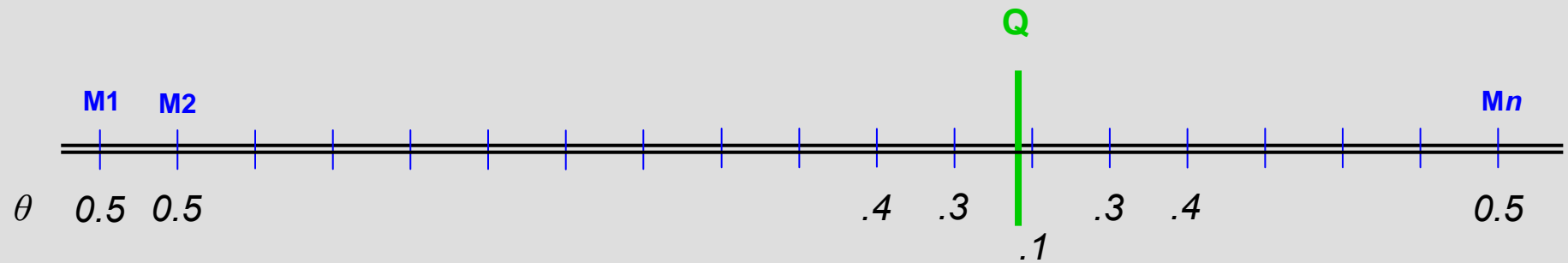


► Overall LOD score for a given  $\theta$  is the sum of all family LOD scores at  $\theta$

eg. LOD=3 for  $\theta=0.28$

# Parametric Linkage Analysis - summary

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- ▶ For each marker, estimate the  $\theta$  that yields highest LOD score across all families
- ▶ This  $\theta$  (and the LOD) will depend upon the mode of inheritance assumed  
MOI determines the genotype at the trait locus Q and thus determines the number of meiosis which are recombinant or nonrecombinant. Limited to Mendelian diseases.
- ▶ Markers with a significant parametric LOD score ( $>3$ ) are said to be linked to the trait locus with recombination fraction  $\theta$

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## 5. Nonparametric Linkage Analysis

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# Approach

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▶ Parametric: genotype marker locus & genotype trait locus

(latter inferred from phenotype according to a specific disease model)

Parameter of interest:  $\theta$  between marker and trait loci

▶ Nonparametric: genotype marker locus & phenotype

If a trait locus truly regulates the expression of a phenotype, then two relatives with similar phenotypes should have similar genotypes at a marker in the vicinity of the trait locus, and vice-versa.

Interest: correlation between phenotypic similarity and marker genotypic similarity

No need to specify mode of inheritance, allele frequencies, etc...

# Phenotypic similarity between relatives

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▶ Squared trait differences

$$(X_1 - X_2)^2$$

▶ Squared trait sums

$$(X_1 + X_2)^2$$

▶ Trait cross-product

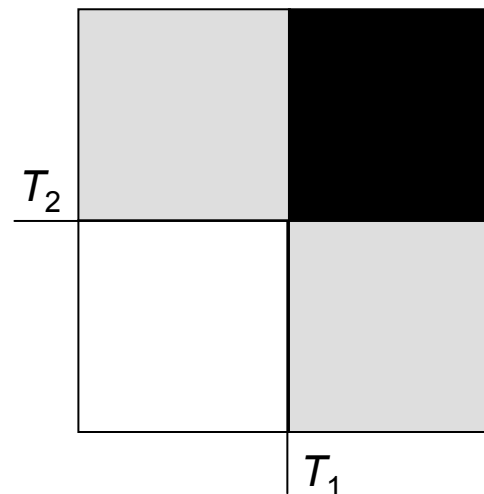
$$[(X_1 - \mu) \cdot (X_2 - \mu)]$$

▶ Trait variance-covariance matrix

$$\begin{Bmatrix} Var(X_1) & Cov(X_1X_2) \\ Cov(X_1X_2) & Var(X_2) \end{Bmatrix}$$

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▶ Affection concordance

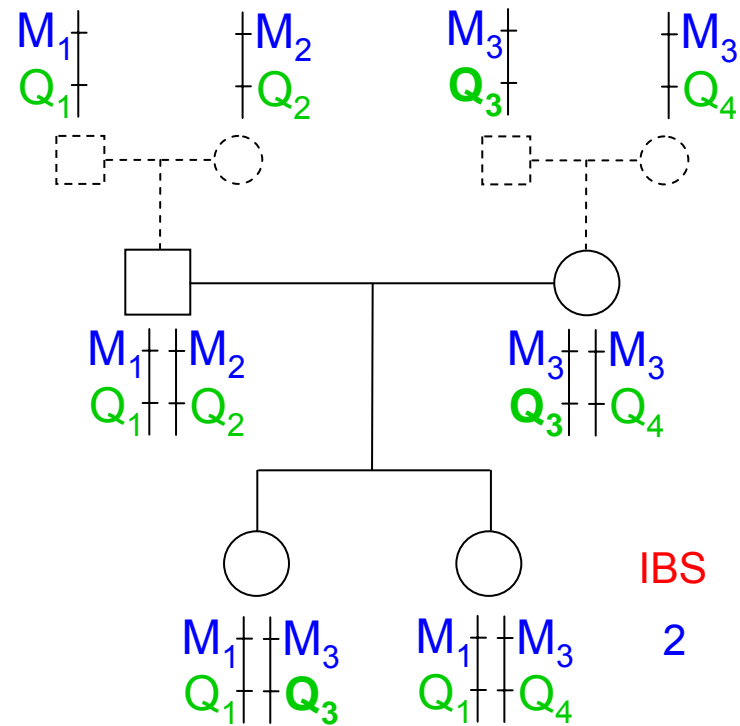




# Genotypic similarity between relatives

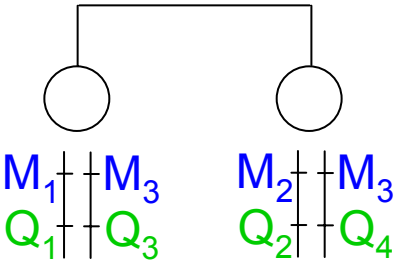
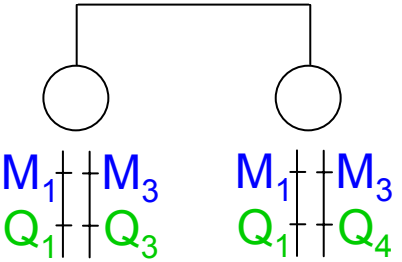
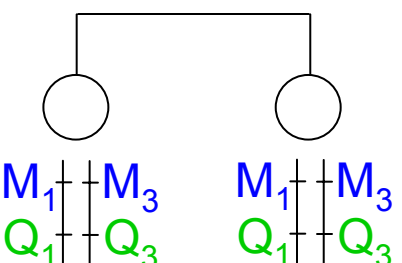
▶ IBS Alleles shared Identical By State “look the same”, may have the same DNA sequence but they are not necessarily derived from a known common ancestor

▶ IBD Alleles shared Identical By Descent are a copy of the same ancestor allele



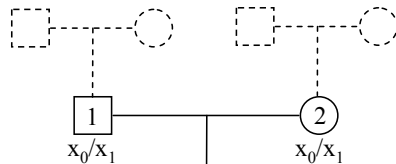
Inheritance vector (M) 0 0 0 1 → 1

# Genotypic similarity between relatives - $\pi$

	Inheritance vector (M)	Number of alleles shared IBD	Proportion of alleles shared IBD - $\pi$
	$\underline{0}$ $\underline{0}$ $\underline{1}$ $\underline{1}$	0	0
	$\underline{0}$ $\underline{0}$ $\underline{0}$ $\underline{1}$	1	0.5
	$\underline{0}$ $\underline{0}$ $\underline{0}$ $\underline{0}$	2	1

# Genotypic similarity between relatives - $\hat{\pi}$

A B C D



$2^{2n}$

		Inheritance vector	IBD
$x_0/x_0$	$x_0/x_0$	0000	2
$x_0/x_0$	$x_0/x_1$	0001	1
$x_0/x_0$	$x_1/x_0$	0010	1
$x_0/x_0$	$x_1/x_1$	0011	0
$x_0/x_1$	$x_0/x_0$	0100	1
$x_0/x_1$	$x_0/x_1$	0101	2
$x_0/x_1$	$x_1/x_0$	0110	0
$x_0/x_1$	$x_1/x_1$	0111	1
$x_1/x_0$	$x_0/x_0$	1000	1
$x_1/x_0$	$x_0/x_1$	1001	0
$x_1/x_0$	$x_1/x_0$	1010	2
$x_1/x_0$	$x_1/x_1$	1011	1
$x_1/x_1$	$x_0/x_0$	1100	0
$x_1/x_1$	$x_0/x_1$	1101	1
$x_1/x_1$	$x_1/x_0$	1110	1
$x_1/x_1$	$x_1/x_1$	1111	2

P (IBD=0)  
P (IBD=1)  
P (IBD=2)

$$\hat{\pi} =$$

# Practical

- ▷ **Aim** (1) Estimate IBD with MERLIN; (2) IBD estimation can be influenced by genotyped individuals and allele frequencies; (3) compute  $\hat{\pi}$

H:\manuel - Copy folder "Linkage" to C:\

1. Open with Notepad: pr1.ped pr1.dat pr1.map pr1.freq
2. Start>Run>C:/Linkage/pfe32.exe
3. Run Command Prompt
4. Keep a File Explorer window open

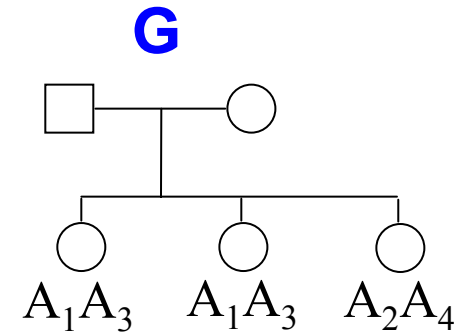
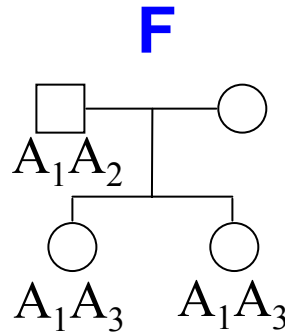
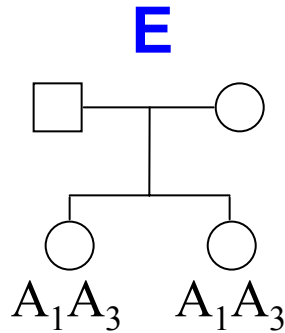
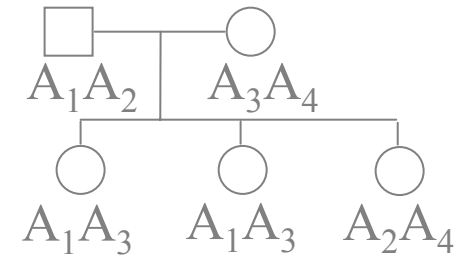
## Exercise1

- (1) Estimate IBD for pedigrees **A**, **B** and **C** in the previous slide
- (2) Change allele frequencies (pr1.freq) from 0.25 0.25 0.25 0.25 to
  - (i) 0.45 0.25 0.25 0.05 and
  - (ii) 0.05 0.25 0.25 0.45

# Practical

## Exercise 2

(1) Modify pr1.ped and estimate IBD probabilities and  $\hat{\pi}$  between twin 1 and twin 2 for pedigrees **E**, **F** and **G**:



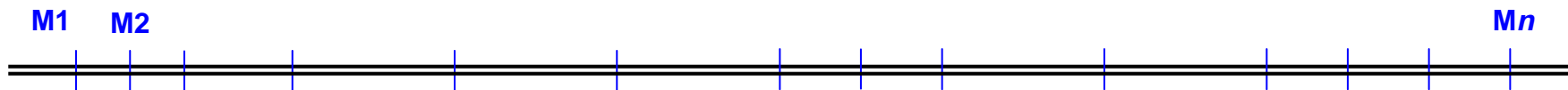
$P(\text{IBD}=0)$

$P(\text{IBD}=1)$

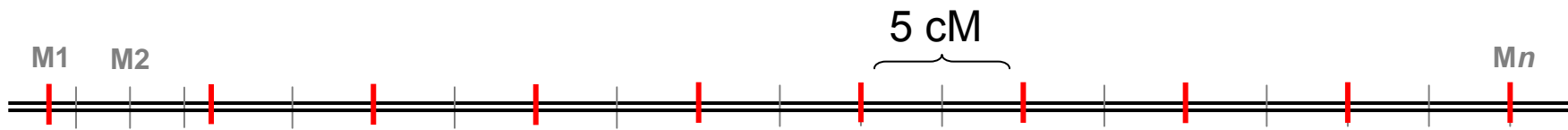
$P(\text{IBD}=2)$

$\hat{\pi}$

Allele frequencies on pr1.freq: 0.25 0.25 0.25 0.25

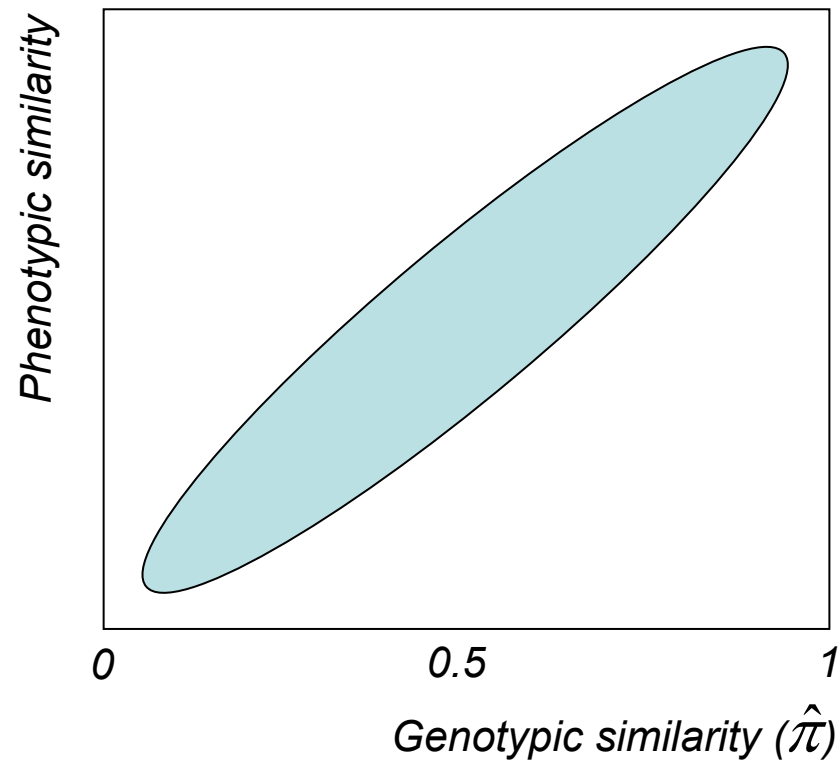


↑  
**IBD at a marker**  
Singlepoint IBD



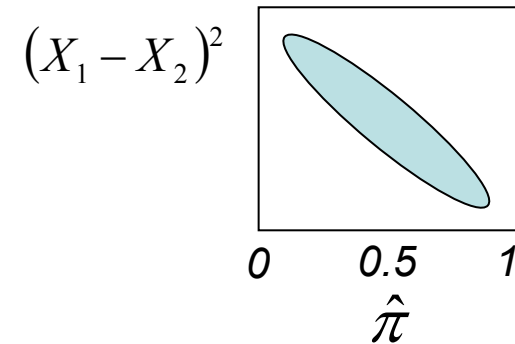
↑  
**IBD at a 'grid'**  
Multipoint IBD

Statistics that incorporate both phenotypic and genotypic similarities



# Haseman-Elston regression – Quantitative traits

$$\begin{aligned}
 & E[(X_1 - X_2)^2 | \hat{\pi}] \\
 &= E[(X_1^2 + X_2^2 - 2 \cdot X_1 \cdot X_2) | \hat{\pi}] \\
 &= \text{Var}(X_1) + \text{Var}(X_2) - 2\text{Cov}(X_1 X_2 | \hat{\pi})
 \end{aligned}$$



$$\begin{aligned}
 \text{Var}(X_1) &= \text{Var}(X_2) = V_Q + V_A + V_C + V_E \\
 \text{Cov}(X_1, X_2 | \hat{\pi}) &= \hat{\pi} \cdot V_Q + \frac{1}{2} \cdot V_A + V_C
 \end{aligned}$$

	$X_1$	$X_2$	$(X_1 - X_2)^2$	$\hat{\pi}$
1	2.2	2.1	0.01	0.9
2	1.9	2.3	0.16	0.6
3	2.3	2.6	0.09	0.7
4	3.4	1.6	3.24	0.1
5	2.5	2.3	0.04	0.8
...				
1000	2.4	2.4	0	0.9

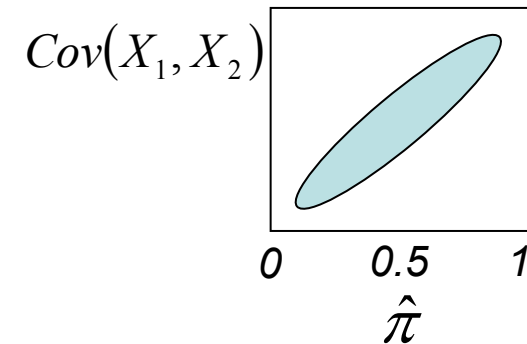
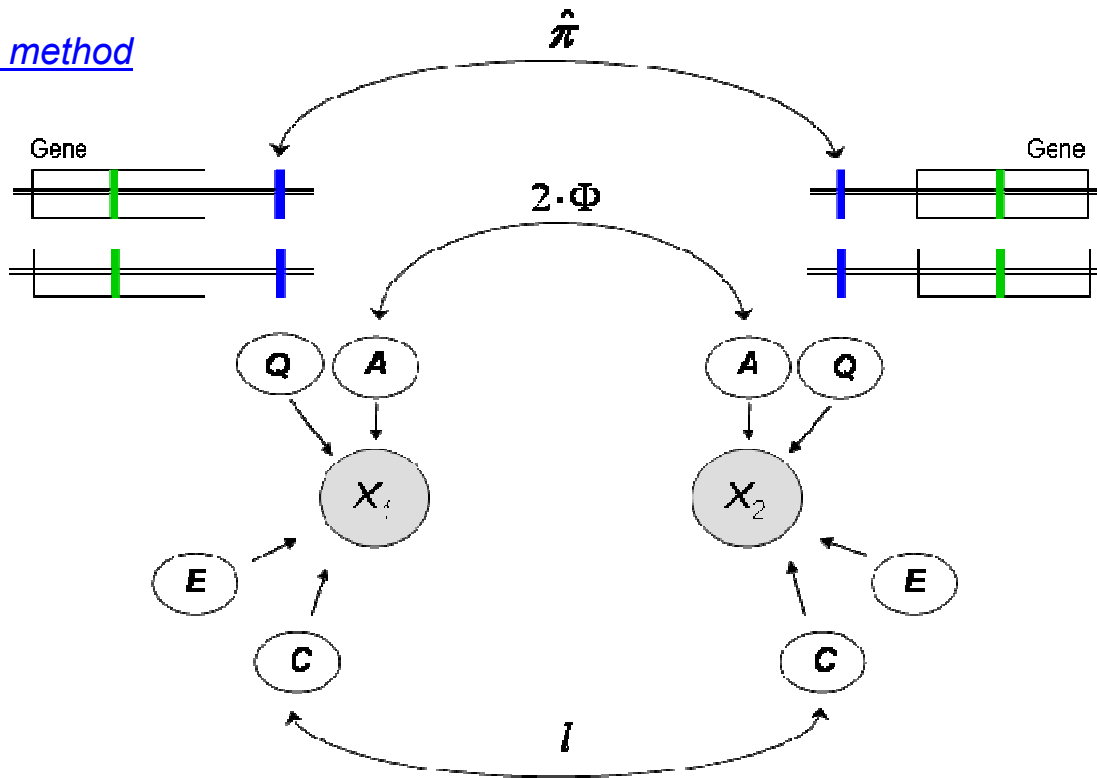
$$E[(X_1 - X_2)^2 | \hat{\pi}] = -2 \cdot V_Q \cdot \hat{\pi} + \underbrace{2 \cdot V_Q + V_A + 2 \cdot V_E}_c$$

Phenotypic dissimilarity =  $b \times$  Genotypic similarity +  $c$



# VC ML – Quantitative & Categorical traits

$\hat{\pi}$  method



$$H_1: \text{Var}(X_1) = \text{Var}(X_2) = V_Q + V_A + V_C + V_E$$

$$\text{Cov}(X_1, X_2 | \hat{\pi}) = \hat{\pi} \cdot V_Q + 2 \cdot \Phi \cdot V_A + l \cdot V_C$$

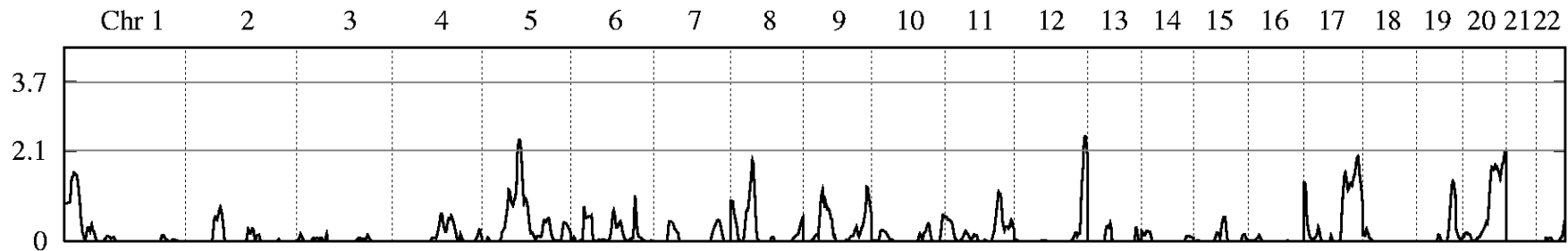
$$LOD = \log_{10} \frac{L(H_1)}{L(H_0)}$$

$$H_0: \text{Var}(X_1) = \text{Var}(X_2) = V_A + V_C + V_E$$

$$\text{Cov}(X_1, X_2 | \hat{\pi}) = 2 \cdot \Phi \cdot V_A + l \cdot V_C$$

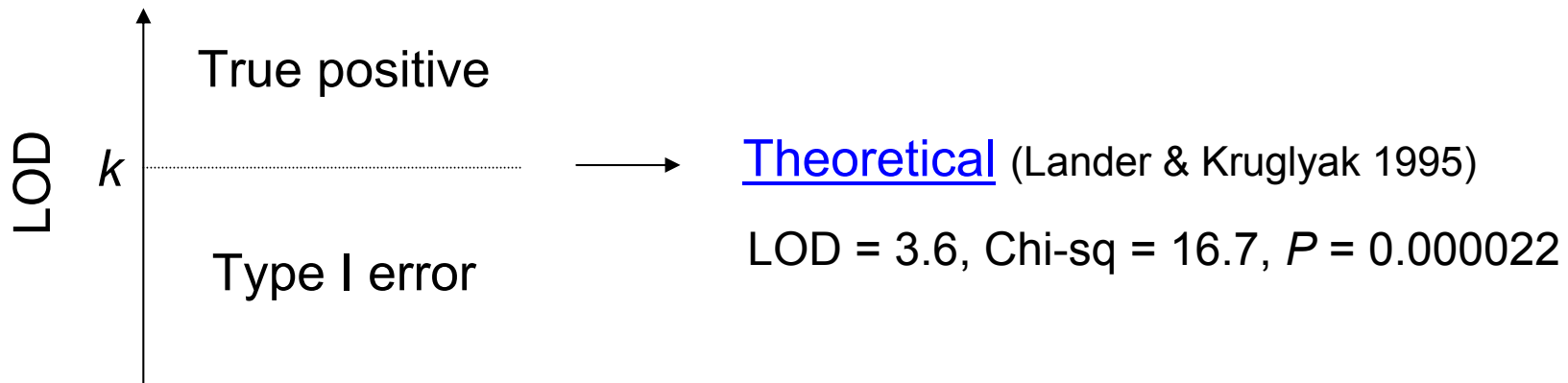
e.g.  $LOD=3$

# Genome-wide linkage analysis (e.g. VC)



Individual LOD scores can be expressed as  $P$  values (Pointwise)

$$\begin{array}{ccccc}
 \text{LOD} & \xrightarrow{(x4.6)} & \text{Chi-sq (n-df)} & \longrightarrow & P \text{ value} \\
 2.1 & & 9.67 & & 0.0009
 \end{array}$$



# Nonparametric Linkage Analysis - summary

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- ▶ No need to specify mode of inheritance
- ▶ Models phenotypic and genotypic similarity of relatives
- ▶ Expression of phenotypic similarity, calculation of IBD
- ▶ HE and VC are the most popular statistics used for linkage of quantitative traits
- ▶ Other statistics available, specially for affection traits