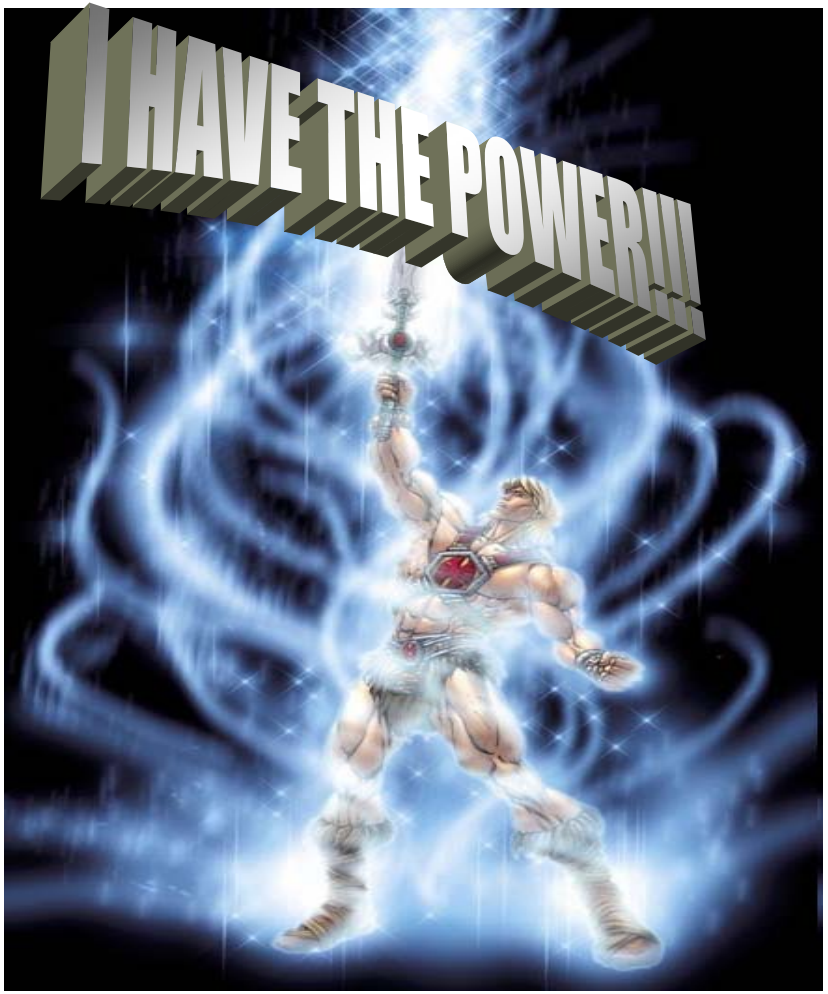
Power and Sample Size



Boulder 2006

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To Be Accomplished

- Introduce concept of power via correlation coefficient (ρ) example
- Identify relevant factors contributing to power
- Practical:
  - Empirical power analysis for univariate twin model (simulation)
  - How to use mx for power

Simple example

Investigate the linear relationship (ρ)

between two random variables X and Y:
$\rho=0$ vs. $\rho\neq0$ (correlation coefficient).

- draw a sample, measure X,Y
- calculate the measure of association ρ (Pearson product moment corr. coeff.)
- test whether $\rho \neq 0$.

How to Test $\rho \neq 0$

- assumed the data are normally distributed
- defined a null-hypothesis ($\rho = 0$)
- chosen α level (usually .05)
- utilized the (null) distribution of the test statistic associated with $\rho=0$
- $t=\rho \sqrt{[(N-2)/(1-\rho^2)]}$

How to Test $\rho \neq 0$

- Sample N=40
- r=.303, t=1.867, df=38, p=.06 α=.05
- As p > α, we fail to reject $\rho = 0$

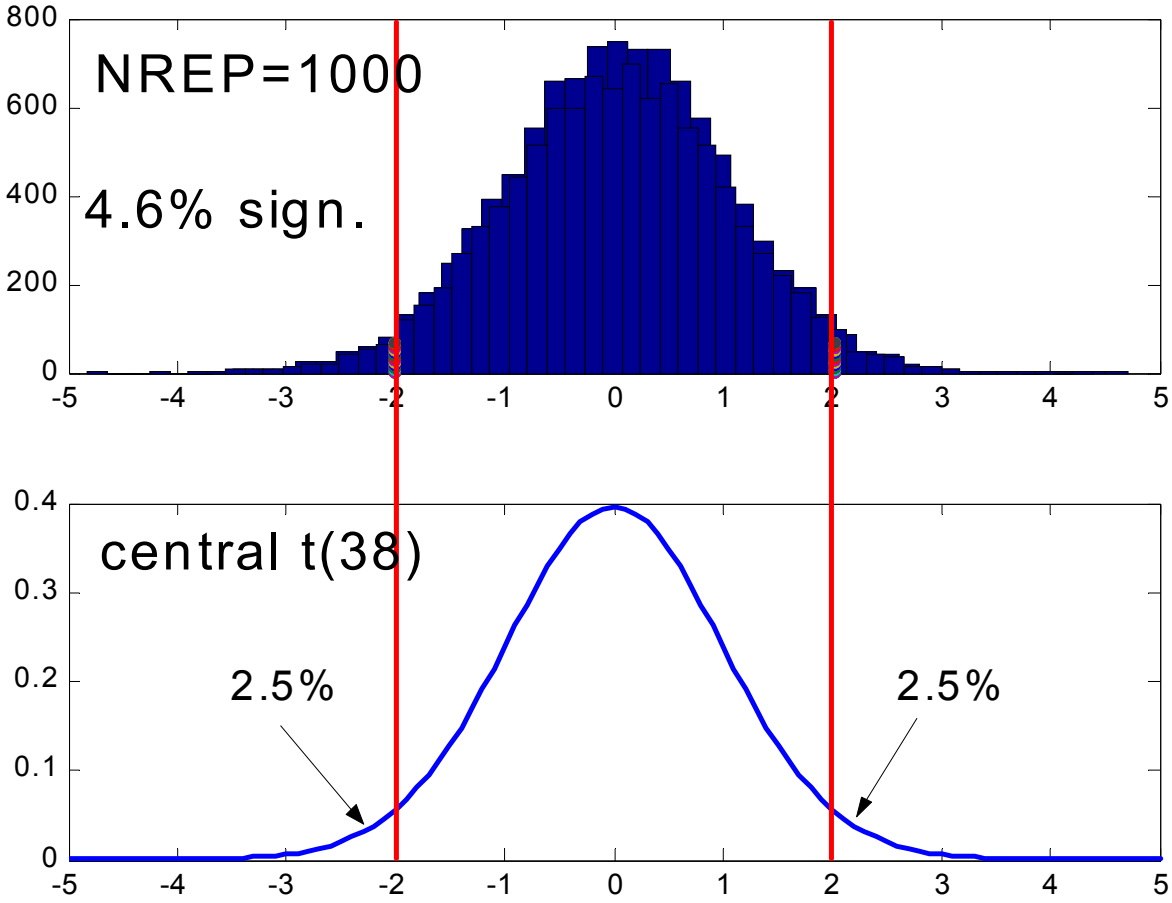
have we drawn the correct conclusion?

α= type I error rate
probability of deciding $\rho \neq 0$
(while in truth ρ=0)

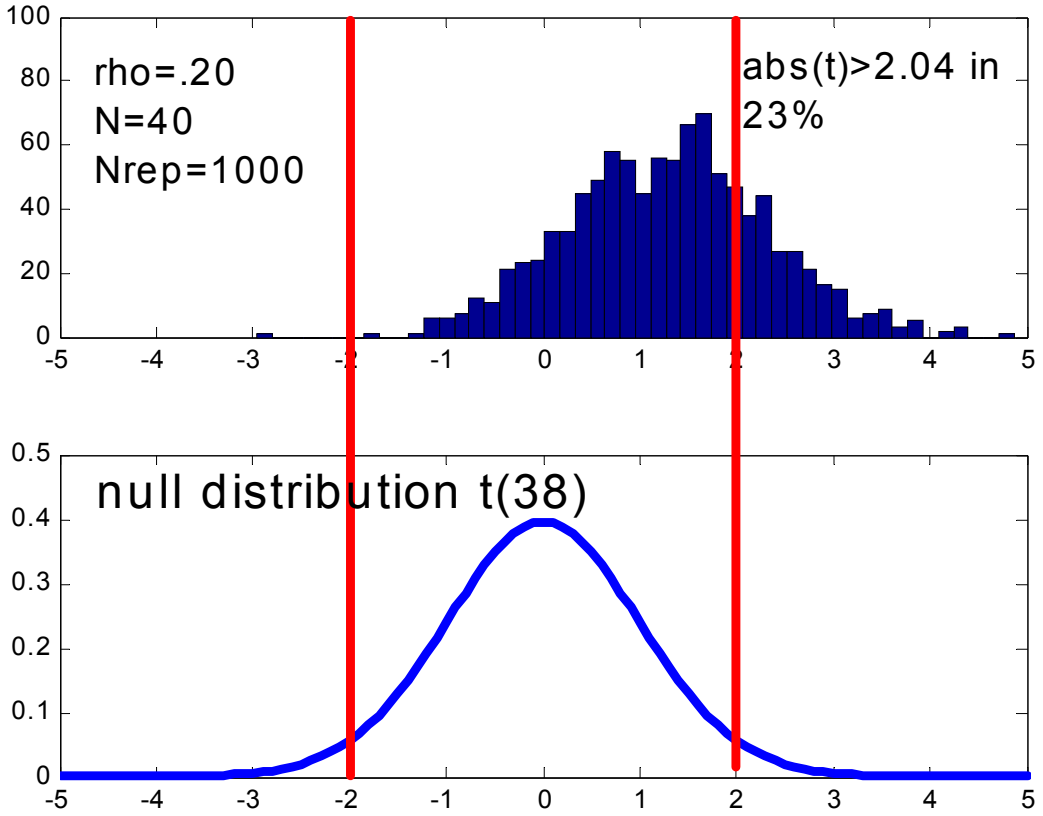
α is often chosen to equal
.05...why?

DOGMA

N=40, r=0, nrep=1000 – central t(38),
α=0.05 (critical value 2.04)



Observed non-null distribution (ρ=.2) and null distribution



In 23% of tests of ρ=0, |t|>2.024 (α=0.05), and thus draw the correct conclusion that of rejecting $\rho = 0$.

The probability of rejecting the null-hypothesis (ρ=0) correctly is 1-β, or the power, when a true effect exists

Hypothesis Testing

- Correlation Coefficient hypotheses:
  - h_o (null hypothesis) is $\rho=0$
  - h_a (alternative hypothesis) is $\rho \neq 0$
    - Two-sided test, where $\rho > 0$ or $\rho < 0$ are one-sided
- Null hypothesis usually assumes no effect
- Alternative hypothesis is the idea being tested

Summary of Possible Results

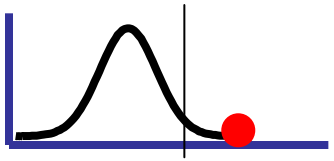
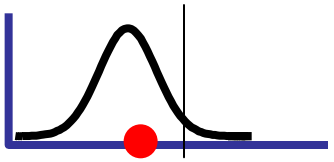
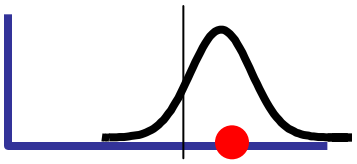
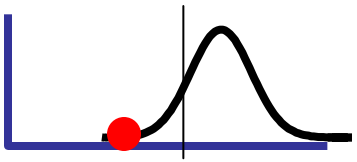
	H-0 true	H-0 false
accept H-0	$1-\alpha$	β
reject H-0	α	$1-\beta$

α=type 1 error rate

β=type 2 error rate

$1-\beta$=statistical power

STATISTICS

REALITY	Rejection of H_0	Non-rejection of H_0
H_0 true	Type I error at rate α 	Nonsignificant result $(1-\alpha)$ 
H_A true	Significant result $(1-\beta)$ 	Type II error at rate β 

Power

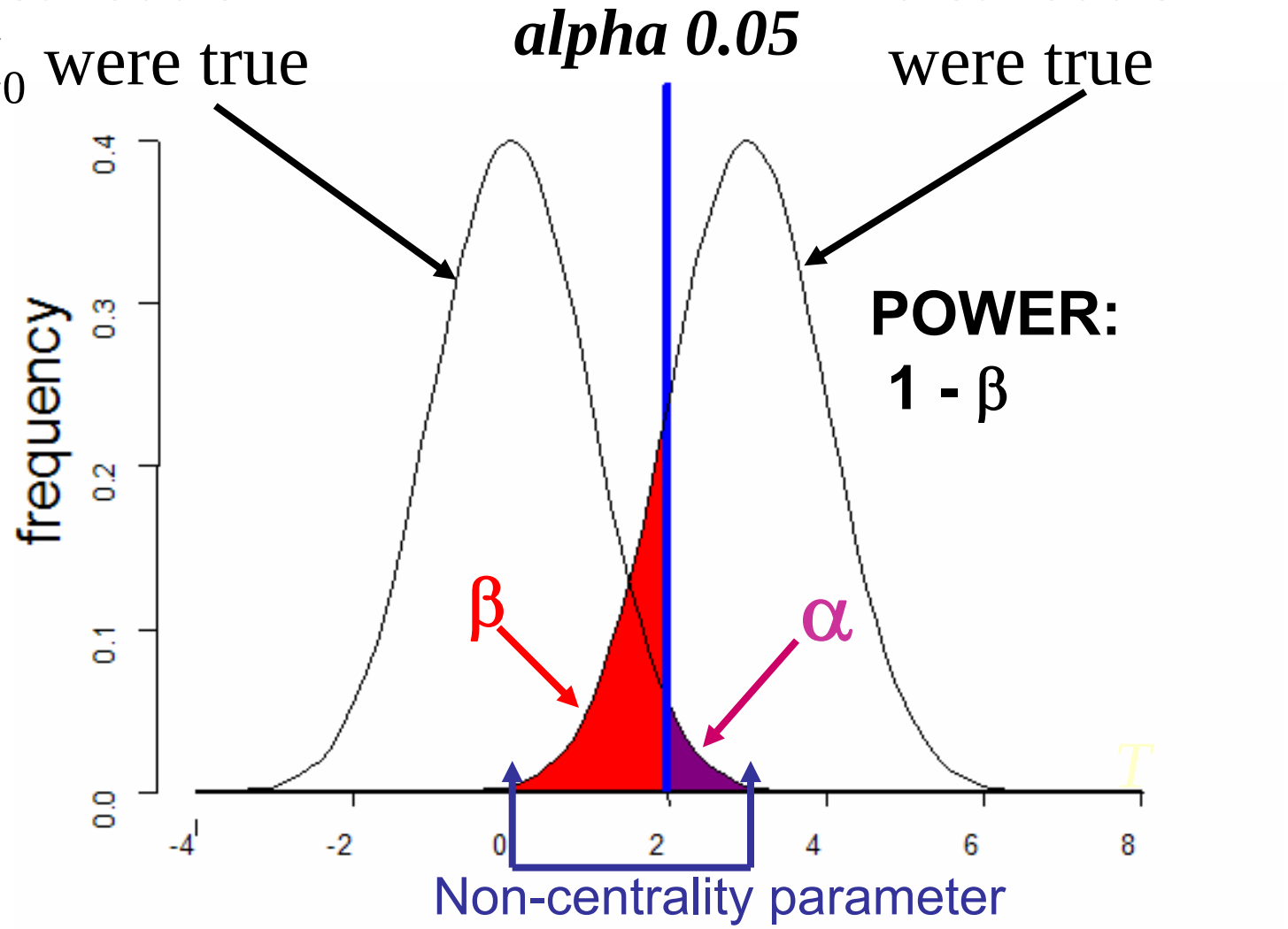
- The probability of rejection of a false null-hypothesis depends on:
  - –the significance criterion (α)
  - –the sample size (N)
  - –the effect size (Δ)

"The probability of detecting a given effect size in a population from a sample of size N, using significance criterion α"

Standard Case

Sampling distribution if H_0 were true

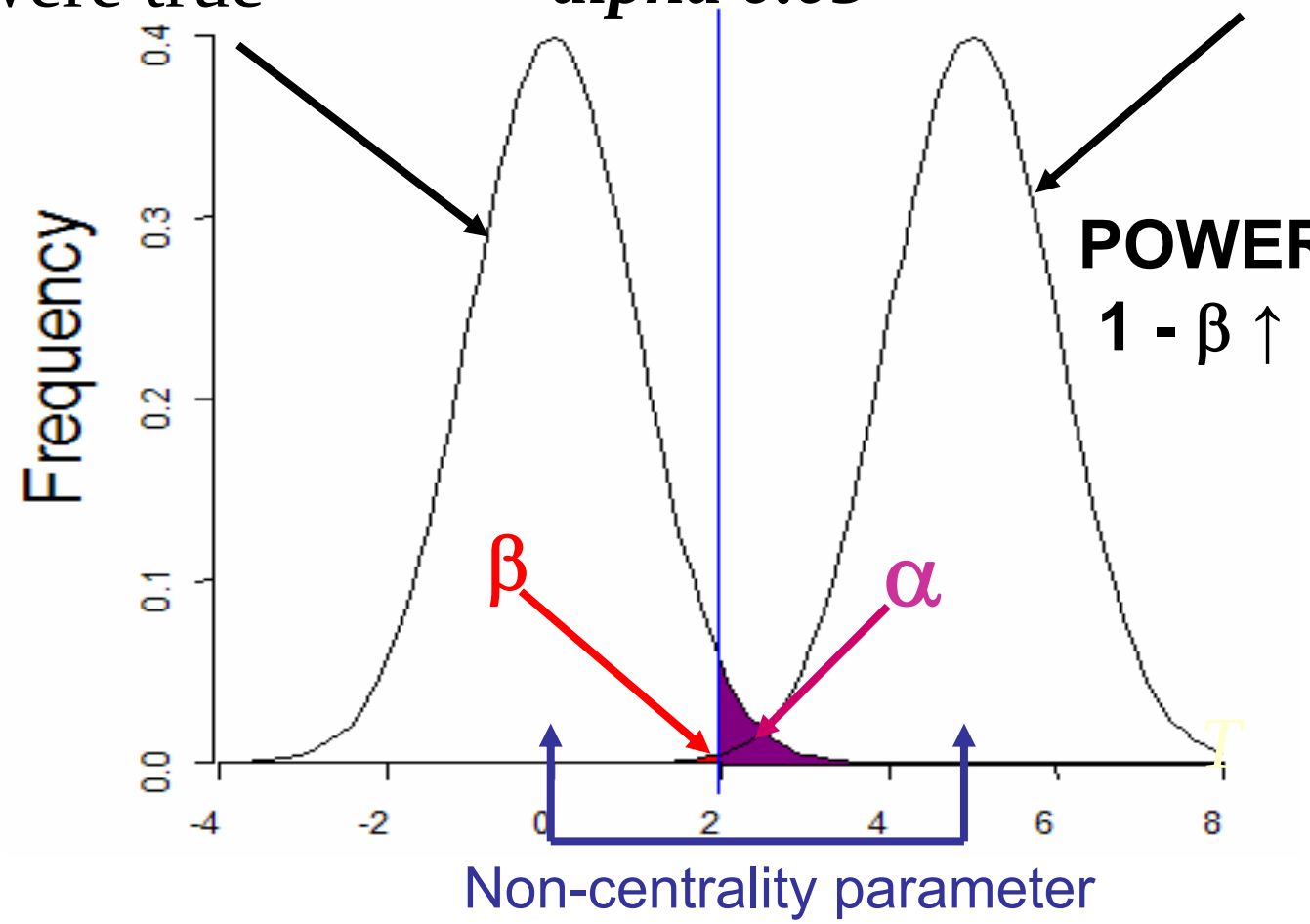
Sampling distribution if H_A were true



Increased effect size

Sampling distribution if H_0 were true

Sampling distribution if H_A were true

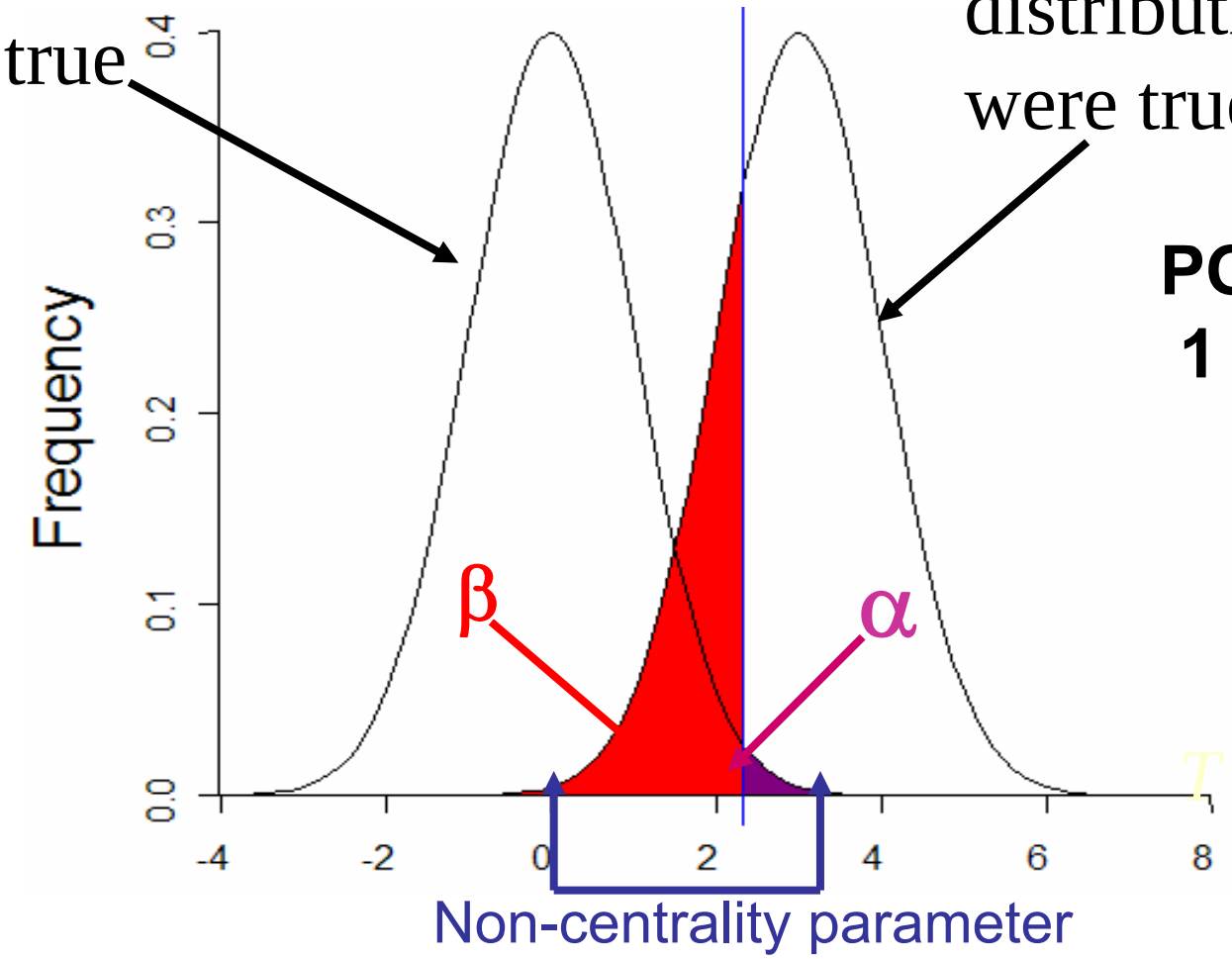


Impact of more conservative

Sampling distribution if H_0 were true

alpha 0.01

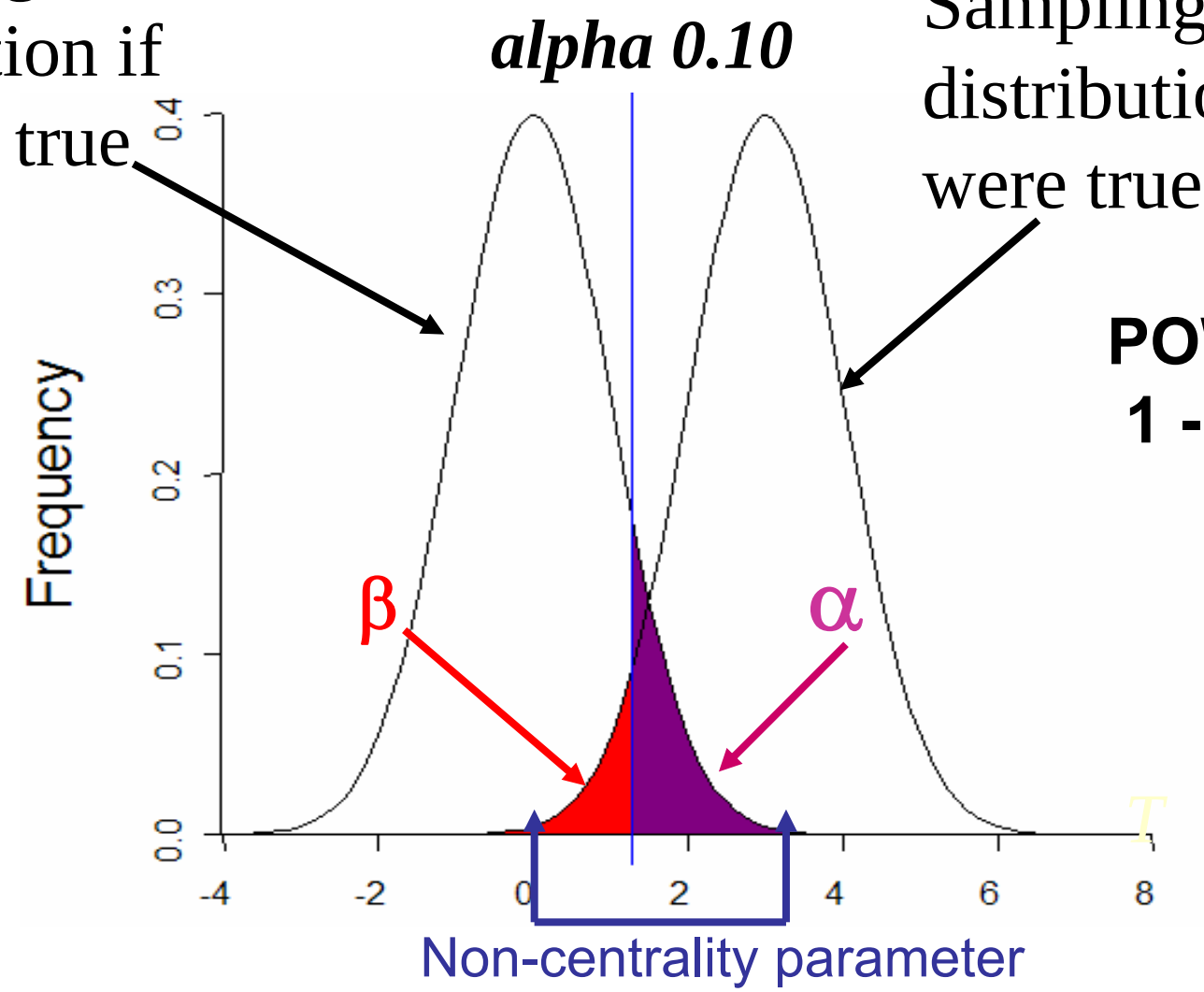
Sampling distribution if H_A were true



POWER:
1 - β ↓

Impact of less conservative

Sampling distribution if H_0 were true



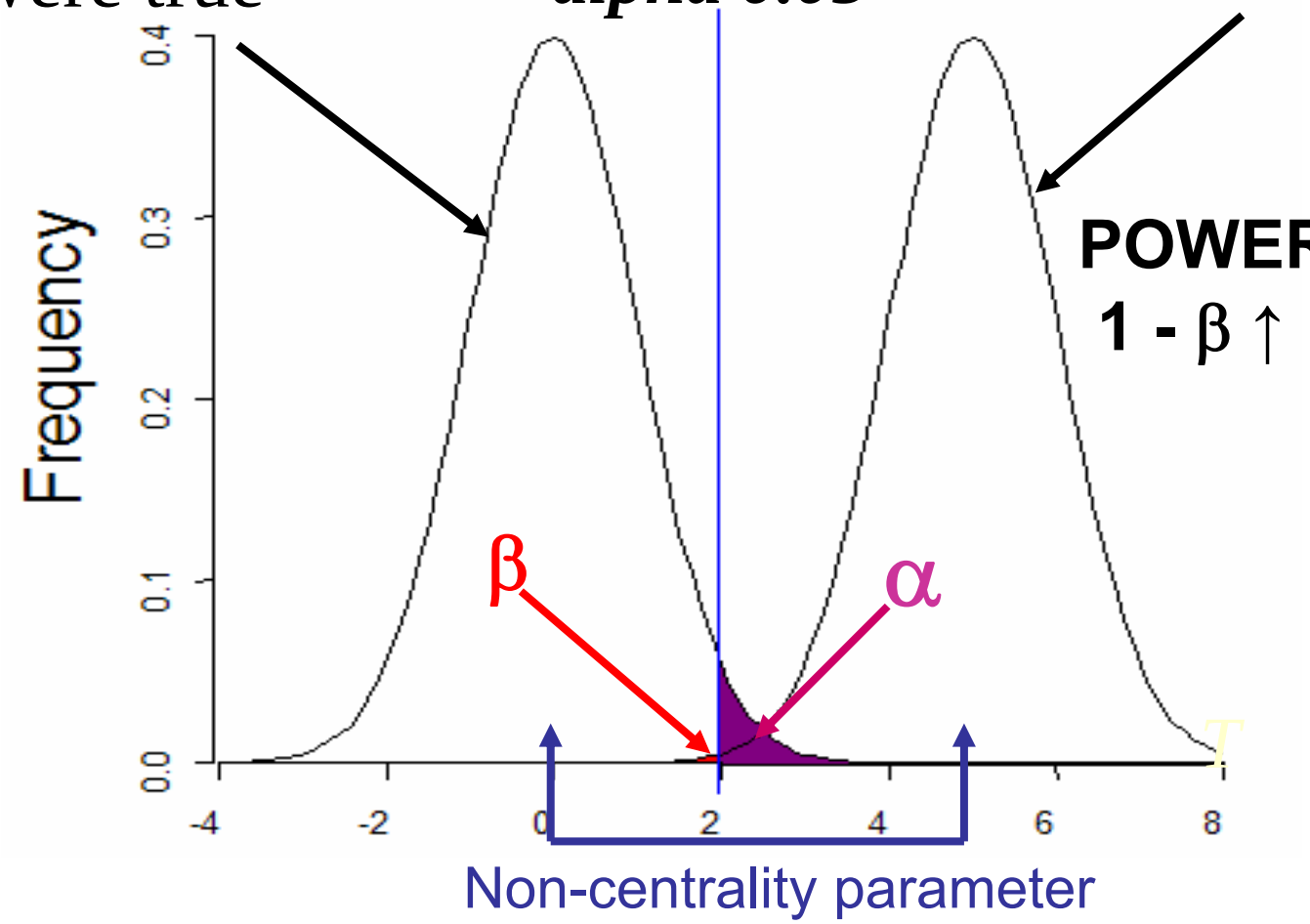
Sampling distribution if H_A were true

POWER:
1 - β ↑

Increased sample size

Sampling distribution if H_0 were true

Sampling distribution if H_A were true



Effects on Power Recap

- Larger Effect Size
- Larger Sample Size
- Alpha Level shifts <Beware the False Positive!!!>
- Type of Data:
  - Binary, Ordinal, Continuous
- Multivariate analysis
- Empirical significance/permutation

When To Do Power Calculations?

- Generally study planning stages of study
- Occasionally with negative result
- No need if significance is achieved
- Computed to determine chances of success