

Linear growth curve models

Modeling individual trajectories over time

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Twin Workshop, Boulder 2008

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models

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Motivation

Modeling
trajectories
Illustration

Approach

Use CFA model
CFA as a LGM
Illustration

Model extensions

Possible
extensions of the
LGM

Extension: ACE
decomposition of
intercept and
slope

ACE for i
ACE for s

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Shape of the trajectories

- say we have a single variable observed at 4 time points
 - glasses of alcohol per week, or other drug consumption
 - numbers of items solved correctly (during a head start or some other training program)
 - sum score of a scale (...)
- interest in modeling the tendency over time
 - scores generally increase/decrease over time
- shape of trajectories can be linear, curvilinear
- latent growth models (LGM's) designed to estimate average trajectory

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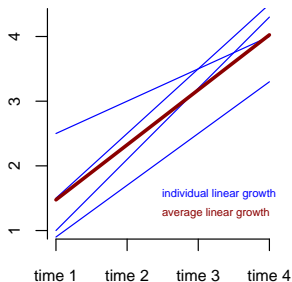
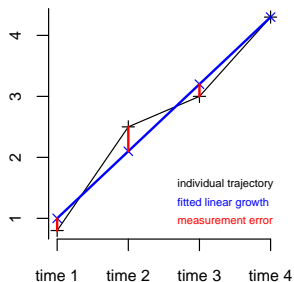
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Difference with other developmental models

- assumption that there is a general structure over time
- individuals allowed to vary, but assumed to follow the general structure
 - e.g., linear increase over time
- this is a very restrictive set of assumptions, resulting a simple developmental model
- the assumptions have to makes sense for your data
- plot the data!
 - usually a subset of your data

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Role of the factors

- let's consider linear growth
- individuals differ with respect to initial value (intercept)
 - intercept is a random variable: varies across individuals
- individuals differ with respect to steepness of the linear growth (slope)
 - slope is a random variable
- intercept and slope of a given individual is not observed but latent
- latent growth models (LGM's) use two factors to capture random intercepts and random slopes
 - factor scores = 'scores' on intercept, slope

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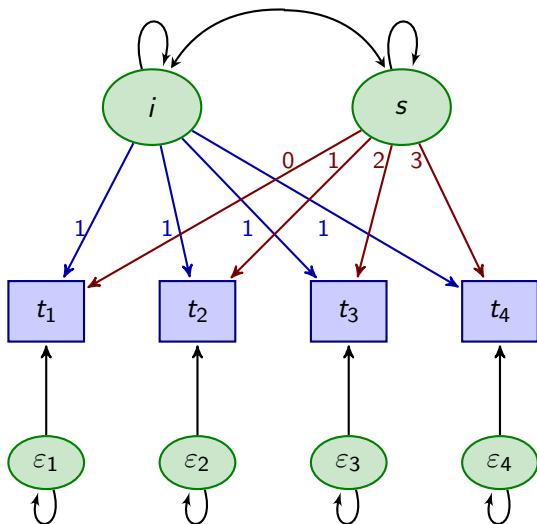
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The common factor model as a growth model



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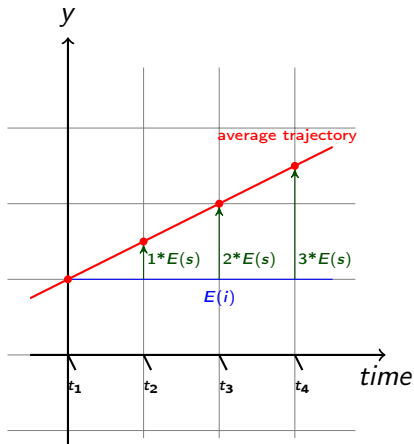
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Intercept and slope factor



The linear growth model is a 2-factor model.

If we calculate the expected value of the observations at each time point, and draw a line through these points, the result is the average trajectory in the sample.

The expectation of the observations at each time point are determined by the average intercept $E(i)$ and the average slope $E(s)$.

$$E(Y_{t1}) = 1 \times E(i) + 0 \times E(s)$$

$$E(Y_{t2}) = 1 \times E(i) + 1 \times E(s)$$

$$E(Y_{t3}) = 1 \times E(i) + 2 \times E(s)$$

$$E(Y_{t4}) = 1 \times E(i) + 3 \times E(s)$$

Practical 1: Fitting linear growth to aggression

Irene...

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Multi-group, multi-variate,..., twins!

- you might want to compare the developmental trajectories of males and females
- growth model can be extended to multiple groups
 - recall: you need to establish measurement invariance if you want to compare groups with respect to factors
- growth model can also be extended to multivariate data at each time point
 - instead of a single variable at each time point, we have a factor at each time point
 - allows for testing measurement invariance over time and across groups
- the model can also be extended to account for heterogeneity
 - growth mixture models
- here: extension to twin data

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Decomposing initial status (1)

- here: we use the simple linear growth model
 - one observed variable over time
- ACE model aims at estimating heritability of
 - initial status
 - linear increase (or decrease)
- recall that in the growth model
 - initial status is the constant baseline over time
 - slope is the linear increase (or decrease) that is added to the constant baseline
- the initial status represented by the factor i does not contain measurement error
- decomposing i is therefore different from decomposing Y at time 1

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Decomposing initial status (2)

- ACE model for the factor i is a common pathway model
- we want to estimate heritability of the growth factors i and s
 - i and s differ across subjects, have variance
 - MZ twins might be more similar with respect to initial status
 - and/or with respect to the slope than DZ twins
- independent pathway model does not decompose the variance of the factors
- therefore, the independent pathway model is not our choice here

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- we want to estimate heritability of the growth factors i and s
 - i and s differ across subjects, have variance
 - MZ twins might be more similar with respect to initial status
 - and/or with respect to the slope than DZ twins
- independent pathway model does not decompose the variance of the factors
- therefore, the independent pathway model is not our choice here

Growth curve models

Lubke, Rebollo, Neale

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Approach

Use CFA model
CFA as a LGM
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Model extensions

Possible extensions of the LGM

Extension: ACE decomposition of intercept and slope

ACE for i
ACE for s

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Decomposing linear growth

- decompose the variance of the factor s using the ACE common pathway model
- heritability of the increase/decrease over time
 - for instance progress of learning (... something)
 - or increase of aggression during puberty
- interesting conceptually
- a important consideration concerning the variance of s :
 - the variance can be small
 - check the saturated model!

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Decomposing the covariance between i and s

- would it make sense on a conceptual level?
 - if the variance of s is small, the estimate of the covariance might not be trustworthy
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Practical 3: ACE decomposition of i and s

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Summary

- LGM's aim at modeling individual trajectories over time
- uses factors to represent random intercepts and random slopes
 - can be extended to curvilinear growth
- second order growth model permits investigating MI over time
- the growth model is set up for a relatively small number of time points
 - large numbers of time points can lead to numerical problems
- decompose the variance of the factors i and s using the ACE common pathway model
- heritability of initial status (= baseline) and increase/decrease over time

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