Multivariate Threshold Models

Specification in Mx

```
#define nsib 2    ! Number of variables * number of siblings = 2
#define maxth 2   ! Maximum number of thresholds
#define nvar 2    ! Number of variables
#define ndef 1    ! Number of definition variables
#define nthr 4    ! nsib x nvar
#NGROUPS 8

G1: MZ Females
Data NInput=8
Ordinal File=data.dat
Labels
famID zyg covar_a covar_b var1_a var2_a var1_b var2_b
Select if zyg = 1 /
 SELECT covar_a covar_b var1_a var2_a var1_b var2_b /
 DEFINITION_VARIABLE covar_a covar_b /

BEGIN MATRICES;
X Lower nvar nvar Free     ! Genetic paths
Y Lower nvar nvar Free     ! Common environmental paths
Z Lower nvar nvar Free     ! Unique environmental paths
H Full 1 1
T Full maxth nthr Free     ! Thresholds
B Full nvar ndef Free      ! Regression betas
L lower maxth maxth        ! For converting incremental to cumulative thresholds
G Full maxth 1             ! For duplicating regression betas across thresholds
K Full ndef nsib           ! Contains definition variables
END MATRICES;
```

Threshold model for multivariate, multiple category data with definition variables:

Part 2   Part 1

$$\mathbf{L} * \mathbf{T} + \mathbf{G} \otimes ((vec(\mathbf{B} * \mathbf{K}))')$$

We will break the algebra into two parts:
1 - Definition variables;
2 - Uncorrected thresholds;
and go through it in detail.

$$\mathbf{L} * \mathbf{T} + \mathbf{G} \otimes \left((vec(\mathbf{B} * \mathbf{K}))' \right)$$

Definition variables

$$\mathbf{B} * \mathbf{K} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} * \begin{matrix} \text{Twin 1} & \text{Twin 2} \\ \begin{bmatrix} sex_A & sex_B \\ age_A & age_B \end{bmatrix} \end{matrix}$$

$$= \begin{bmatrix} b_{11} \times sex_A + b_{12} \times age_A & b_{11} \times sex_B + b_{12} \times age_B \\ b_{21} \times sex_A + b_{22} \times age_A & b_{21} \times sex_B + b_{22} \times age_B \end{bmatrix}$$

Threshold correction
Twin 1
Variable 1

Threshold correction
Twin 2
Variable 1

Threshold correction
Twin 1
Variable 2

Threshold correction
Twin 2
Variable 2

$$\mathbf{L} * \mathbf{T} + \mathbf{G} \otimes ((vec(\mathbf{B} * \mathbf{K}))')$$

$$vec(\mathbf{B} * \mathbf{K}) = \begin{bmatrix} b_{11} \times sex_A + b_{12} \times age_A \\ b_{21} \times sex_A + b_{22} \times age_A \\ b_{11} \times sex_B + b_{12} \times age_B \\ b_{21} \times sex_B + b_{22} \times age_B \end{bmatrix}$$

Transpose:

$$\begin{bmatrix} b_{11} \times sex_A + b_{12} \times age_A & b_{21} \times sex_A + b_{22} \times age_A & b_{11} \times sex_B + b_{12} \times age_B & b_{21} \times sex_B + b_{22} \times age_B \end{bmatrix}$$

$$\mathbf{L} * \mathbf{T} + \boxed{\mathbf{G} \otimes \left(\left(vec(\mathbf{B} * \mathbf{K})\right)'\right)}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} b_{11}sex_A + b_{12}age_A & b_{21}sex_A + b_{22}age_A & b_{11}sex_B + b_{12}age_B & b_{21}sex_B + b_{22}age_B \end{bmatrix}$$

$$= \begin{bmatrix} b_{11}sex_A + b_{12}age_A & b_{21}sex_A + b_{22}age_A & b_{11}sex_B + b_{12}age_B & b_{21}sex_B + b_{22}age_B \\ b_{11}sex_A + b_{12}age_A & b_{21}sex_A + b_{22}age_A & b_{11}sex_B + b_{12}age_B & b_{21}sex_B + b_{22}age_B \end{bmatrix}$$

$$\mathbf{L} * \mathbf{T} + \mathbf{G} \otimes ((vec(\mathbf{B} * \mathbf{K}))')$$

$$\mathbf{L} * \mathbf{T} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{13} \\ t_{21} & t_{22} & t_{23} & t_{24} \end{bmatrix}$$

$$= \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{11} + t_{21} & t_{12} + t_{22} & t_{13} + t_{23} & t_{14} + t_{24} \end{bmatrix}$$

Thresholds 1 & 2
Twin 2
Variable 2

Thresholds 1 & 2
Twin 1
Variable 1

Thresholds 1 & 2
Twin 1
Variable 2

Thresholds 1 & 2
Twin 2
Variable 1

$$\mathbf{L} * \mathbf{T} + \mathbf{G} \otimes ((vec(\mathbf{B} * \mathbf{K}))')$$

$$\begin{bmatrix} t_{11} & t_{12} & \cdots \\ t_{11} + t_{21} & t_{12} + t_{22} & \cdots \end{bmatrix} + \begin{bmatrix} b_{11}sex_A + b_{12}age_A & b_{11}sex_B + b_{12}age_B & \cdots \\ b_{11}sex_A + b_{12}age_A & b_{11}sex_B + b_{12}age_B & \cdots \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} t_{11} + b_{11}sex_A + b_{12}age_A & t_{12} + b_{21}sex_A + b_{22}age_A & \cdots \\ t_{11} + t_{21} + b_{11}sex_A + b_{12}age_A & t_{12} + t_{22} + b_{21}sex_A + b_{22}age_A & \cdots \end{bmatrix}$$