Matrix Algebra Exercises

Let $\mathbf{A} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ $\mathbf{D} = \begin{bmatrix} 1 & 3 \\ 2 & .5 \\ 3 & 6 \end{bmatrix}$
$\mathbf{E} = \begin{bmatrix} 1 & .2 \\ .2 & 1 \end{bmatrix}$ $\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Find:

1. $\mathbf{AA}'$
2. $\mathbf{A}'\mathbf{A}$
3. $\mathbf{AA}'$
4. $\mathbf{A} + \mathbf{B}$
5. $\mathbf{A} + \mathbf{B}'$ [1]
6. $\mathbf{A} + \mathbf{2B}$
7. $\mathbf{CC}'$
8. $\mathbf{DD}'$
9. $\det(\mathbf{E})$
10. $\det(\mathbf{F})$
11. $\log(\det(\mathbf{E}))$
12. $\mathbf{EF^{-1}}$
13. $\mathrm{trace}(\mathbf{EF^{-1}})$
14. $100\left[\log(\det(\mathbf{E})) - \log(\det(\mathbf{F})) + \mathrm{trace}(\mathbf{EF}^{-1}) - 2\right]$

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[1] This may be a trick question!

Likelihood Exercises

Let $\mathbf{x} = \begin{bmatrix} .5 & -.3 \end{bmatrix}$ $\mu' = \begin{bmatrix} 0 & 0 \end{bmatrix}$ $\mathbf{\Sigma} = \begin{bmatrix} 1 & .5 \\ .5 & 1 \end{bmatrix}$ The vector $\mathbf{x}$ represents the data vector for a pair of twins each measured on one variable. Vector μ represents the population mean vector and $\mathbf{\Sigma}$ the population covariance matrix. μ and $\mathbf{\Sigma}$ are parameters which might be estimated for a sample. Here we just take them as fixed values.

(a) Compute the normal theory likelihood of vector $\mathbf{x}$. What does this likelihood represent geometrically?

(b) Compute -2 times the log-likelihood (normal theory) of the data vector $\mathbf{x}$.