Liability Threshold Models

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Aims

- Introduce model fitting to categorical data
 - Define liability and describe assumptions of the liability model
 - Show how heritability of liability can be estimated from categorical twin data
- Selection
- Practical exercise

Ordinal data

Measuring instrument discriminates between two or a few ordered categories

-Absence (0) or presence (1) of a disorder -Score on a Q item e.g. : 0 - 1, 0 - 4

A single ordinal variable

Assumptions:

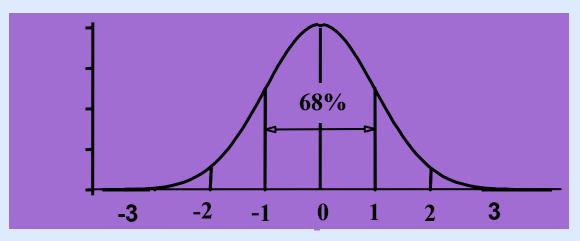
- (1) Underlying normal distribution of liability
- (2) The liability distribution has

1 or more thresholds (cut-offs)

The standard Normal distribution

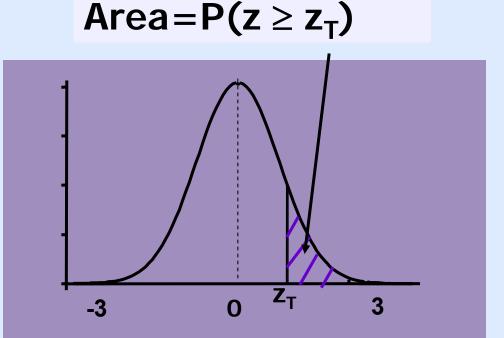
Liability is a *latent* variable, the scale is arbitrary, distribution is, therefore, assumed to be the *Standard Normal Distribution* (SND) or z-distribution:

- mean $(\mu) = 0$ and SD $(\sigma) = 1$
- z-values are the number of SD away from the mean
- area under curve translates directly to probabilities
 - > Stand Normal Probability Density function (Φ)



Standard Normal Cumulative Probability in right-hand tail (For negative z values, areas are found by symmetry)

 Z_0 Area 0 .50 50% .2 .42 42% .4 .35 35% .6 .27 27% .21 8. 21% 1 .16 16% 1.2 .12 12% 1.4 8% 80. 1.6 6% .06 1.8 .036 3.6% .023 2.3% 2 .014 1.4% 2.2 2.4 .008 .8% .005 .5% 2.6 2.8 .003 .3% .2% .002 2.9



$$\int_{Z_T}^{\infty} \Phi(L_1; \mu = 0, \sigma^2 = 1) dL_1$$

Example: single dichotomous variable

It is possible to find a **z-value** (threshold) so that the proportion exactly matches the observed proportion of the sample e.g. sample of 1000 individuals, where 120 have met the criteria for a disorder (12%): the z-value is 1.2

Z ₀	Area		_		
.6 .8 1 1.2 1.4 1.6	.27 .21 .16 .12 .08 .055	27% 21% 16% 12% 8% 6%			
1.8 2	.036 .023	3.6% 2.3%		-3 0 1	2 3
2.2	.023	1.4%			
2.4	.008	.8%		/	N
2.6	.005	.5%	lln	affected (0)	Affected (1)
2.8	.003	.3%			Allected (1)
2.9	.002	.2%	Counts:	880	120

Two ordinal variables: Data from twin pairs

> Contingency Table with 4 observed cells:

cell a: pairs concordant for unaffected cell d: pairs concordant for affected cell b/c: pairs discordant for the disorder

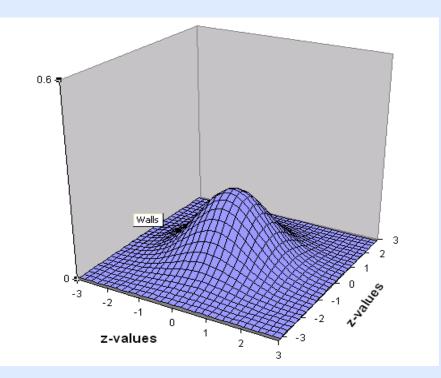
Twin1		
Twin2	0	1
0	٩	b
1	С	d

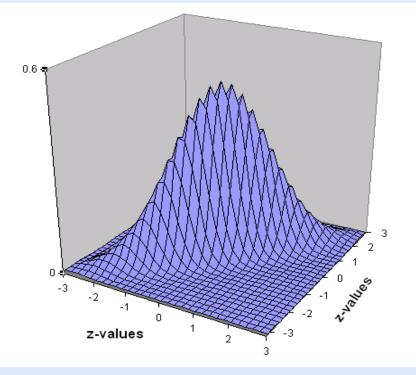
0 = unaffected 1 = affected

Joint Liability Model for twin pairs

- Assumed to follow a bivariate normal distribution, where both traits have a mean of 0 and standard deviation of 1, but the correlation between them is unknown.
- The shape of a bivariate normal distribution is determined by the correlation between the traits

Bivariate Normal

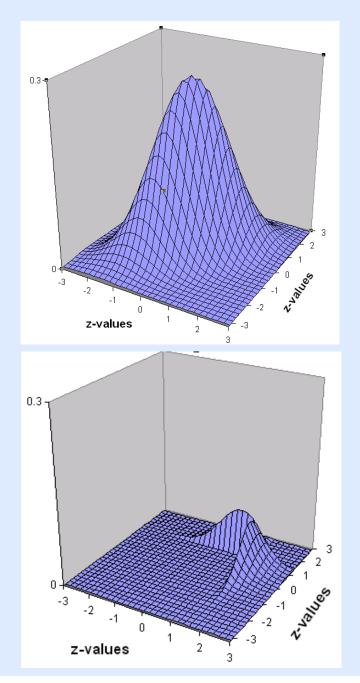


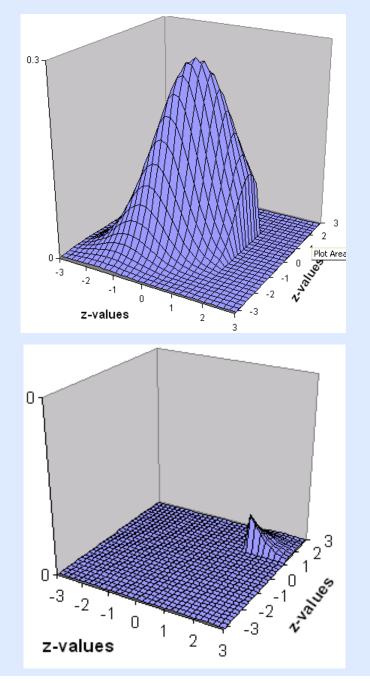


r =.00

r = .90

Bivariate Normal (R=0.6) partitioned at threshold 1.4 (z-value) on both liabilities





Expected proportions

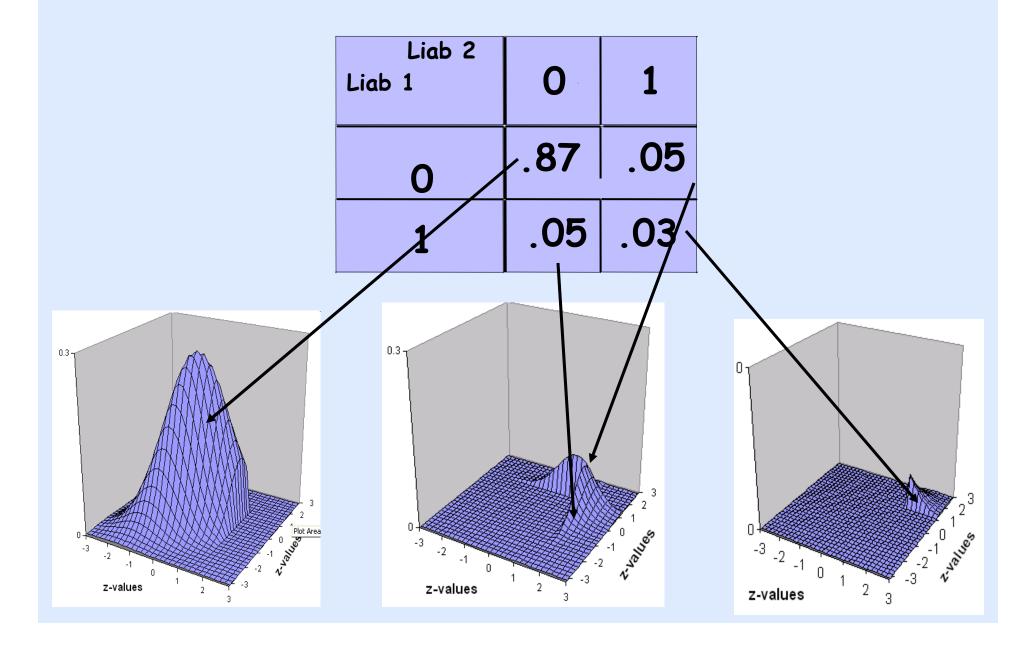
By **numerical integration** of the bivariate normal over two dimensions: the liabilities for twin1 and twin2

e.g. the probability that both twins are affected :

$$\int_{T_1}^{\infty} \int_{T_2}^{\infty} \Phi(L_1, L_2; \mathbf{0}, \boldsymbol{\Sigma}) dL_1 dL_2$$

 Φ is the bivariate normal probability density function, L₁ and L₂ are the liabilities of twin1 and twin2, with means 0, and Σ is the correlation matrix of the two liabilities T₁ is threshold (z-value) on L₁, T₂ is threshold (z-value) on L₂

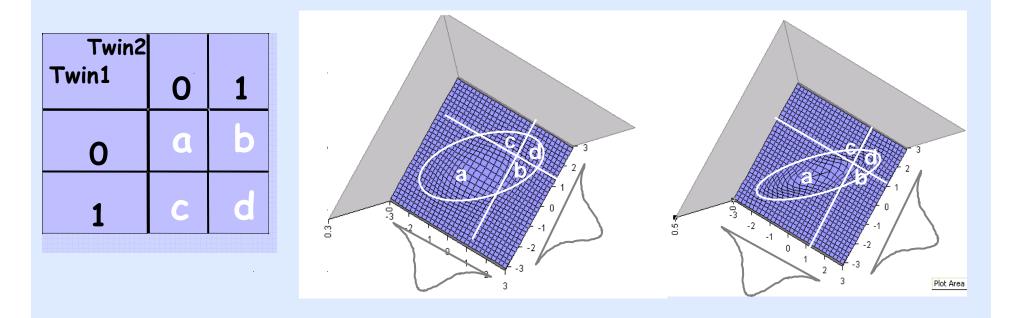
Expected Proportions of the BN, for R = 0.6, $T_1 = 1.4$, $T_2 = 1.4$



How can we estimate correlations from CT?

The correlation (shape) of the BN and the two thresholds determine the relative proportions of observations in the 4 cells of the CT.

Conversely, the sample proportions in the 4 cells can be used to estimate the correlation and the thresholds.

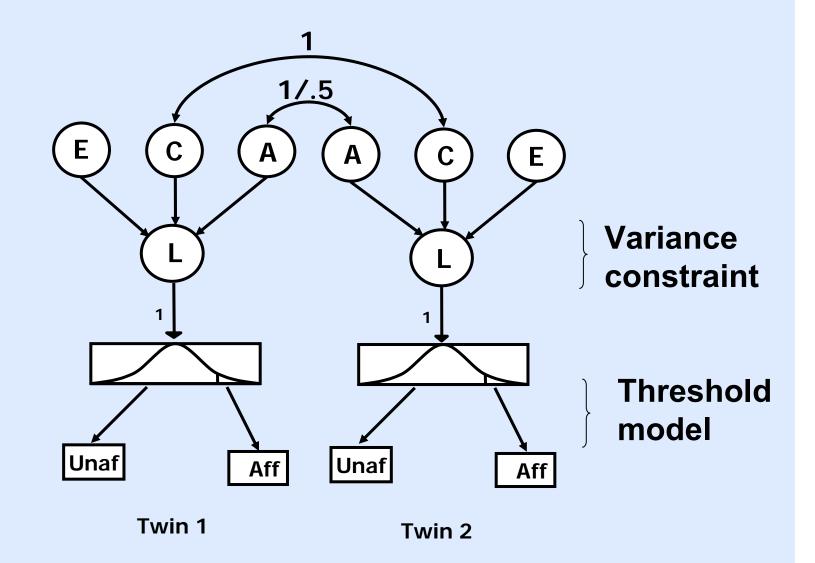


The classical Twin Model

- Estimate correlation in liability separately for MZ and DZ
- A variance decomposition (A, C, E) can be applied to *liabilities*

→ estimate of the heritability of the *liability*

ACE Liability Model



Summary

- To estimate correlations for categorical traits (counts) we make assumptions about the joint distribution of the data (Bivariate Normal)
- The relative proportions of observations in the cells of the Contingency Table are translated into proportions under the BN
- The most likely thresholds and correlations are estimated
- Genetic/Environmental variance components are estimated based on these correlations derived from MZ and DZ data

How can we fit ordinal data in Mx?

1. Summary statistics: CT

Mx has a built-in fit function for the maximum likelihood analysis of 2-way Contingency Tables

> analyses limited to only two variables, with 2 or more ordered classes

Tetrachoric	Polyc	horic	
CTable 2 2 40 5 4 10	CT 3 3 39 4 5 6 15 6 4 7 10	CT 3 2 39 13 16 15 14 20	A sinput lines

Fit function

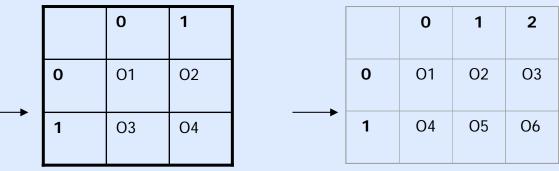
- 1. Mx calculates twice the log-likelihood of the observed frequency data under the model using:
 - Observed frequency in each cell
 - Expected proportion in each cell (Num Integration of the BN)
- 2. Mx calculates the log-likelihood of the observed frequency data themselves
- 3. An approximate χ^2 statistic is then computed by taking the differences in these 2 likelihoods

(Equations given in Mx manual, pg 91-92)

Test of assumption

For a 2x2 CT with 1 estimated TH on each liability, the χ^2 statistic is always 0, 3 observed statistics, 3 param, df=0 (it is always possible to find a correlation and 2 TH to perfectly explain the proportions in each cell). No goodness of fit of the normal distribution assumption.

This problem is resolved if the CT is at least 2x3 (i.e. more than 2 categories on at least one liability) A significant χ^2 reflects departure from normality.



How can we fit ordinal data in Mx?

2. Raw data analyses

- Advantages over CT:
 - multivariate
 - handles missing data
 - moderator variables (for covariates e.g. age)

ORD File=...dat

Mx input lines

Fit function

- The likelihood for a vector of observed ordinal responses is computed by the expected proportion in the corresponding cell of the MV distribution
- The likelihood of the model is the sum of the likelihoods of all vectors of observation
- This is a value that depends on the number of observations and isn't very interpretable (as with continuous data)
- So we compare it with the LL of other models, or a saturated (correlation) model to get a χ^2 model-fit index

(Equations given in Mx manual, pg 89-90)

Raw Ordinal Data File

	ordinal	ordinal
Zyg	respons1	respons2
1	0	0
1	0	0
1	0	1
2	1	0
2 2	0	0
1	1	1
2	•	1
- 2 2	0	•
2	0	1

NOTE: smallest category should always be 0 !!

SORT !

Sorting speeds up computation time
 If i = i+1 then likelihood not recalculated

In e.g. bivariate, 2 category case, there are only 4 possible vectors of observations: 1 1, 0 1, 1 0, 00 and, therefore, only 4 integrals for Mx to calculate if the data file is sorted. **Selection**

Selection

For rare disorders (e.g. Schizophrenia), selecting a random population sample of twins will lead to the vast majority of pairs being unaffected.

A more efficient design is to ascertain twin pairs through a register of affected individuals.

Types of ascertainment

Double (complete) Ascertainment

Single Ascertainment

Twin 1 Twin 2	0	1	
0	00 ^a	01 ^b	
1	10 ^c	11 ^d	

Proband			
Co-twin	0	1	
0	00 ^a	01 ^b	
1	10 ^c	11 ^d	

Ascertainment Correction

When using ascertained samples, the Likelihood Function needs to be corrected.

Omission of certain classes from observation leads to an increase of the likelihood of observing the remaining individuals

Need re-normalization

Mx corrects for incomplete ascertainment by dividing the likelihood by the proportion of the population remaining after ascertainment

Ascertainment Correction in Mx: univariate

Com	plete	Single	
Asce	rtainment	Ascertainment	
СТа	ble 2 2	CTable 2 2	
-1	b	-1 b	
С	d	-1 d	

CT from ascertained data can be analysed in Mx by simply substituting a -1 for the missing cells

 Thresholds need to be fixed > population prevalence of disorder e.g. Schiz (1%), z-value = 2.33

Ascertainment Correction in Mx: multivariate

Write own fit function.

Adjustment of the Likelihood function is accomplished by specifying a user-defined fit function that adjusts for the missing cells (proportions) of the distribution.

A twin study of genetic relationships between psychotic symptoms. Cardno, Rijsdijk, Sham, Murray, McGuffin, Am J Psychiatry. 2002, 159(4):539-45

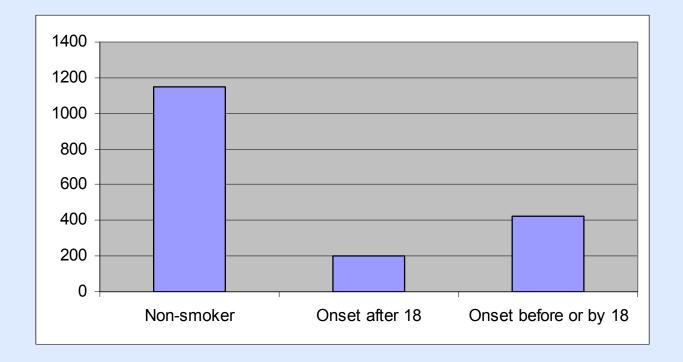
Practical

Sample and Measures

- Australian Twin Registry data (QIMR)
- Self-report questionnaire
 - Non-smoker, ex-smoker, current smoker
 - Age of smoking onset
- Large sample of adult twins
 - + family members
 - Today using MZMs (785 pairs)
 and DZMs (536 pairs)



- Variable: age at smoking onset, including non-smokers
- Ordered as:
 - Non-smokers / late onset / early onset



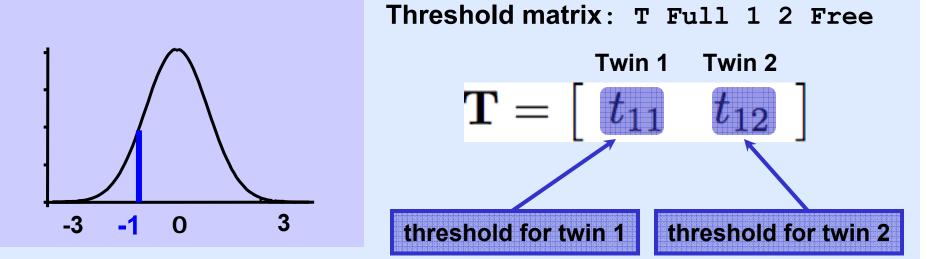
Practical Exercise

Analysis of age of onset data

- Estimate thresholds
- Estimate correlations
- Fit univariate model

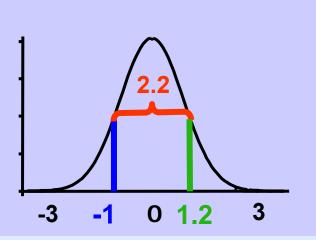
Observed counts from ATR data: MZM DZM Twin 1 Twin 1 0 1 2 0 1 2 0 368 24 46 0 203 22 63 $\frac{1}{2} 1 26 15 21 \frac{1}{2} 17 5 16$ 2 54 22 209 2 65 12 133

Mx Threshold Specification: Binary

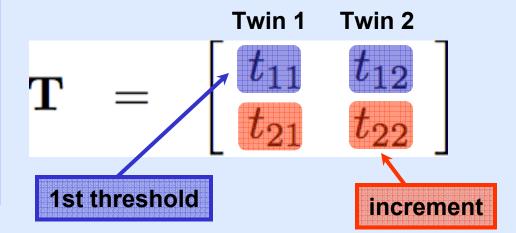


Mx Threshold Model: Thresholds T /

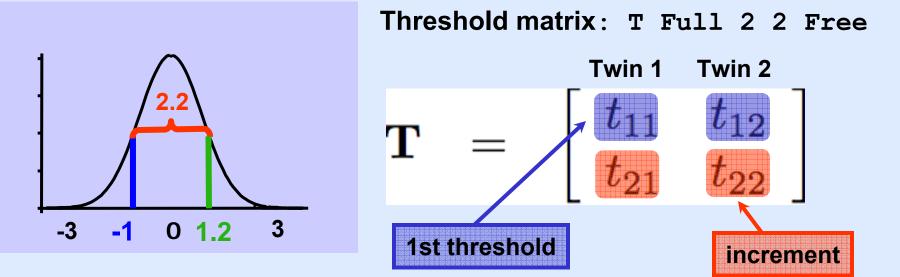
Mx Threshold Specification: 3+ Cat.



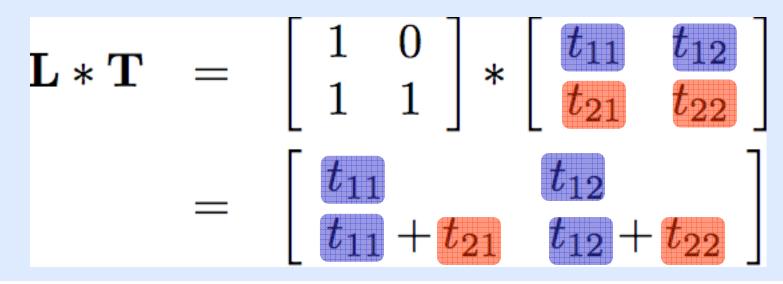
Threshold matrix: T Full 2 2 Free



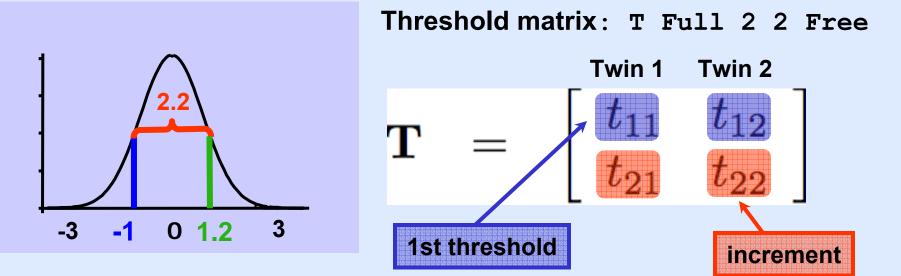
Mx Threshold Specification: 3 + Cat.



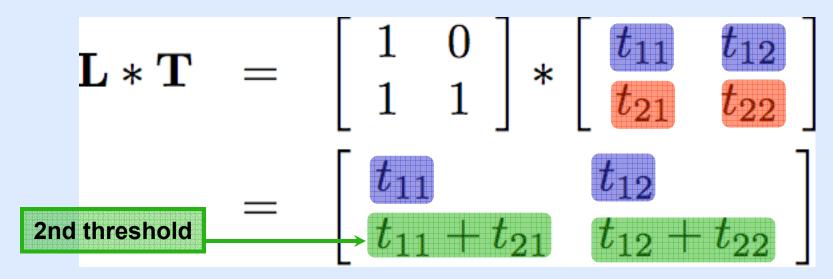
Mx Threshold Model: Thresholds L*T /



Mx Threshold Specification: 3 + Cat.



Mx Threshold Model: Thresholds L*T /



```
#define nvarx2 2
#define nthresh 2
#ngroups 2
```

```
G1: Data and model for MZM correlation
DAta NInput_vars=3
Missing=.
Ordinal File=smk_prac.ord
Labels
    zyg ageon_t1 ageon_t2
SELECT IF zyg = 2
SELECT ageon_t1 ageon_t2 /
Begin Matrices;
R STAN nvarx2 nvarx2 FREE
```

T FULL nthresh nvarx2 FREE L Lower nthresh nthresh

End matrices;

Value 1 L 1 1 to L nthresh nthresh

```
#define nvarx2 2 ! Number of variables x number of twins
#define nthresh 2 ! Number of thresholds=num of cat-1
#ngroups 2
G1: Data and model for MZM correlation
DAta NInput vars=3
Missing=.
Ordinal File=smk_prac.ord ! Ordinal data file
Labels
  zyg ageon t1 ageon t2
SELECT IF zyq = 2
SELECT ageon t1 ageon t2 /
Begin Matrices;
R STAN nvarx2 nvarx2 FREE
T FULL nthresh nvarx2 FREE
L Lower nthresh nthresh
End matrices;
```

Value 1 L 1 1 to L nthresh nthresh

```
#define nvarx2 2  ! Number of variables per pair
#define nthresh 2 ! Number of thresholds=num of cat-1
#ngroups 2
G1: Data and model for MZM correlation
DAta NInput vars=3
Missing=.
Ordinal File=smk prac.ord ! Ordinal data file
Labels
  zyg ageon t1 ageon t2
SELECT IF zyq = 2
SELECT ageon t1 ageon t2 /
Begin Matrices;
R STAN nvarx2 nvarx2 FREE ! Correlation matrix
T FULL nthresh nvarx2 FREE
L Lower nthresh nthresh
End matrices;
```

Value 1 L 1 1 to L nthresh nthresh

```
#define nvarx2 2  ! Number of variables per pair
#define nthresh 2 ! Number of thresholds=num of cat-1
#ngroups 2
G1: Data and model for MZM correlation
DAta NInput vars=3
Missing=.
Ordinal File=smk prac.ord ! Ordinal data file
Labels
  zyg ageon t1 ageon t2
SELECT IF zyq = 2
SELECT ageon t1 ageon t2 /
Begin Matrices;
R STAN nvarx2 nvarx2 FREE ! Correlation matrix
T FULL nthresh nvarx2 FREE ! thresh tw1, thresh tw2
L Lower nthresh nthresh ! Sums threshold increments
End matrices;
```

Value 1 L 1 1 to L nthresh nthresh ! initialize L

COV R / Thresholds L*T / Bound 0.01 1 T 1 1 T 1 2 Bound 0.1 5 T 2 1 T 2 2 Start 0.2 T 1 1 T 1 2 Start 0.2 T 2 1 T 2 2 Start .6 R 2 1 Option RS Option func=1.E-10 END COV R / ! Predicted Correlation matrix for MZ pairs Thresholds L*T / ! Threshold model, to ensure t1>t2>t3 etc. Bound 0.01 1 T 1 1 T 1 2 Bound 0.1 5 T 2 1 T 2 2 Start 0.2 T 1 1 T 1 2 Start 0.2 T 2 1 T 2 2 Start .6 R 2 1 Option RS Option func=1.E-10 END COV R / ! Predicted Correlation matrix for MZ pairs Thresholds L*T / ! Threshold model, to ensure t1>t2>t3 etc. Bound 0.01 1 T 1 1 T 1 2 Bound 0.1 5 T 2 1 T 2 2 !Ensures +ve threshold increment Start 0.2 T 1 1 T 1 2 !Starting value for 1st thresholds Start 0.2 T 2 1 T 2 2 !Starting value for increment Start .6 R 2 1 !Starting value for correlation Option RS Option func=1.E-10 !Function precision less than usual END ! Test equality of thresholds between Twin 1 and Twin 2 EQ T 1 1 1 T 1 1 2 !constrain TH1 equal across Tw1 and Tw2 MZM EQ T 1 2 1 T 1 2 2 !constrain TH2 equal across Tw1 and Tw2 MZM EQ T 2 1 1 T 2 1 2 !constrain TH1 equal across Tw1 and Tw2 DZM EQ T 2 2 1 T 2 2 2 !constrain TH2 equal across Tw1 and Tw2 DZM EQ T 2 2 1 T 2 2 2 !constrain TH2 equal across Tw1 and Tw2 DZM End

Get cor.mxs

! Test equality of thresholds between MZM & DZM EQ T 1 1 1 T 1 1 2 T 2 1 1 T 2 1 2 !TH1 equal across all males EQ T 1 2 1 T 1 2 2 T 2 2 1 T 2 2 2 !TH2 equal across all males End

Estimates: Saturated Model

-2LL	df		Twin 1	Twin 2	Twin 1	Twin 2	
Saturated			М	Z	DZ		
5128.185	3055	Threshold 1	0.09	0.12	0.03	0.05	
		Threshold 2	0.31	0.33	0.24	0.26	
		Correlation	0.81		0.55		

MATRIX R:			MAT	RIX T:		Matrix of	EXPE	CTED thr	esholds	
	1	1 1.0000	2	1	1 0.0865	2	Threshold		GEON_T1A 0.0865	_
2	2		1.0000	2	0.2212		Threshold		0.3078	

Estimates: Saturated Model

-2LL	df		Twin 1	Twin 2	Twin 1	Twin 2	
Saturated			М	7	DZ		
5128.185	3055	Threshold 1	0.09	0.12	0.03	0.05	
		Threshold 2	0.31	0.33	0.24	0.26	
		Correlation	0.81		0.55		

MATRIX R:			MAI	RIX T:		Matrix of EXPECTED thresholds				
	1	2		1	2		AGEON TIAGEON T2			
1	1.0000		1	0.0865	0.1171	Threshold	$1 0.0\overline{8}65 0.1\overline{1}71$			
2	0.8095	1.0000	2	0.2212	0.2153	Threshold	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			

Exercise I

- Fit saturated model
 - Estimates of thresholds
 - Estimates of polychoric correlations
- Test equality of thresholds
 - Examine differences in threshold and correlation estimates for saturated model and sub-models
- Examine correlations
 - What model should we fit?

Raw ORD File: smk_prac.ord Script: polychor_smk.mx Location: Faculty\Fruhling\Categorical_Data

Estimates: Sub-models

X ²	df	Р		Twin 1	Twin 2	Twin 1	Twin 2	
Sub-mode	11	I		Μ	Z	DZ		
			Threshold 1					
			Threshold 2					
			Correlation					
Sub-mode	12			M	Z	DZ		
			Threshold 1					
			Threshold 2					
			Correlation					

Raw ORD File: smk_prac.ord Script: polychor_smk.mx Location: Faculty\Fruhling\Categorical_Data

Estimates: Sub-models

X ²	df	Р		Twin 1	Twin 2	Twin 1	Twin 2		
Sub-mode	11			M	Z	D	DZ		
0.77	4	0.94	Threshold 1	0.10	0.10	0.04	0.04		
			Threshold 2	0.32	0.32	0.25	0.25		
			Correlation	0.81		0.55			
Sub-mode	12			M	Z	DZ			
2.44	6	0.88	Threshold 1	0.07	0.07	0.07	0.07		
			Threshold 2	0.29	0.29	0.29	0.29		
			Correlation	0.81		0.55			

ACEcat_smk.mx

#define nvar 1
#define nvarx2 2
#define nthresh 2
#ngroups 4

!number of variables per twin
!number of variables per pair
!number of thresholds=num of cat-1
!number of groups in script

G1: Parameters for the Genetic model Calculation Begin Matrices; X Low nvar nvar FREE !Additive genetic path coefficient Y Low nvar nvar FREE !Common environmental path coefficient Z Low nvar nvar FREE !Unique environmental path coefficient End matrices; Begin Algebra; A=X*X' : !Additive genetic variance (path X squared) C=Y*Y' ; !Common Environm variance (path Y squared) E=Z*Z'; !Unique Environm variance (path Z squared) End Algebra; start .6 X 1 1 Y 1 1 Z 1 1 !starting value for X, Y, Z Interval @95 A 1 1 C 1 1 E 1 1!requests the 95%CI for h2, c2, e2 End

```
G2: Data and model for MZ pairs
DAta NInput vars=3
Missing=.
Ordinal File=prac smk.ord
Labels
  zyg ageon t1 ageon t2
SELECT IF zyq = 2
SELECT ageon t1 ageon t2 /
Matrices = group 1
T FULL nthresh nvarx2 FREE !Thresholds for twin 1 and twin 2
L Lower nthresh nthresh
                              !Predicted covariance matrix for MZs
COV
 ( A + C + E | A + C
     A + C | A + C + E \rangle /
Thresholds L*T /
                              !Threshold model
Bound 0.01 1 T 1 1 T 1 2
                              !Ensures +ve threshold increment
Bound 0.1 5 T 2 1 T 2 2
Start 0.1 T 1 1 T 1 2
                              !Starting values for the 1st thresholds
Start 0.2 T 1 1 T 1 2
                              !Starting values for increment
Option rs
End
```

```
G3: Data and model for DZ pairs
DAta NInput vars=4
Missing=.
Ordinal File=prac smk.ord
Labels
  zyg ageon t1 ageon t2
SELECT IF zyq = 4
SELECT ageon t1 ageon t2 /
Matrices = group 1
T FULL nthresh nvarx2 FREE
                            !Thresholds for twin 1 and twin 2
L Lower nthresh nthresh
                              !0.5
H FULL 1 1
COVARIANCE
                              !Predicted covariance matrix for DZs
 (A + C + E | H@A + C
  H@A + C | A + C + E ) /
Thresholds L*T /
                              !Threshold model
Bound 0.1 1 T 1 1 T 1 2
                              !Ensures +ve threshold increment
Bound 0.1 5 T 2 1 T 2 2
```

```
!Starting values for the 1st thresholds
!Starting values for increment
```

End

Option rs

Start 0.1 T 1 1 T 1 2

Start 0.2 T 1 1 T 1 2

```
G4: CONSTRAIN VARIANCES OF OBSERVED VARIABLES TO 1

CONSTRAINT

Matrices = Group 1

I UNIT 1 1

CO A+C+E= I / !constrains the total variance to equal 1

Option func=1.E-10

End
```

Constraint groups and degrees of freedom

As the total variance is constrained to unity, we can estimate one VC from the other two, giving us one less independent parameter:

A + C + E = 1 therefore E = 1 - A - C

So each constraint group adds a degree of freedom to the model.

Exercise II

- Fit ACE model
 - What does the threshold model look like?
 - Change it to reflect the findings from exercise I
 - What are the VC estimates?

Raw ORD File: smk_prac.ord Script: ACEcat_smk.mx Location: Faculty\Fruhling\Categorical_Data

Results

Model	-2LL	df	X ²	df	Ρ
Saturated	5128.185	3055			
ACE	5130.628	3061	2.443	6	0.88

3 Confidence intervals requested in group 1

Mat	rix	Elen	nent	Int.	Estimate	Lower	Upper	Lfail	Ufail
A	1	1	1	95.0	0.5254	0.3278	0.7391	0 0	0 0
С	1	1	1	95.0	0.2862	0.0846	0.4659	0 0	0 0
Е	1	1	1	95.0	0.1884	0.1476	0.2376	6 1	0 1