# Pearson-Aitken Selection Formula:

Define a set of random variables, **z**, partitioned such that

$$\mathbf{z} = \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}.$$

It is assumed that the relationship among all variables is linear and homoscedastic.

Denote the mean vector of the variables as

$$\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_x \\ \boldsymbol{\mu}_y \end{pmatrix},$$

and the variance covariance matrix as

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_x & \mathbf{C}_{xy} \\ \mathbf{C}_{yx} & \mathbf{V}_y \end{pmatrix}.$$

Assume that selection changes mean vector $\boldsymbol{\mu}_x$ to $\tilde{\boldsymbol{\mu}}_x$. Then the new mean vector for the **y** variables after selection will equal

$$\tilde{\boldsymbol{\mu}}_y = \boldsymbol{\mu}_y + \mathbf{C}_{yx}\mathbf{V}_x^{-1}(\tilde{\boldsymbol{\mu}}_x - \boldsymbol{\mu}_x).$$

If the selection process changes the covariance matrix among the **x** variables from $\mathbf{V}_x$ to $\tilde{\mathbf{V}}_x$, then the covariance matrix will change from **V** to

$$\tilde{\mathbf{V}} = \begin{pmatrix} \tilde{\mathbf{V}}_x & \tilde{\mathbf{V}}_x\mathbf{V}_x^{-1}\mathbf{C}_{xy} \\ \mathbf{C}_{yx}\mathbf{V}_x^{-1}\tilde{\mathbf{V}}_x & \mathbf{V}_y - \mathbf{C}_{yx}(\mathbf{V}_x^{-1} - \mathbf{V}_x^{-1}\tilde{\mathbf{V}}_x\mathbf{V}_x^{-1})\mathbf{C}_{xy} \end{pmatrix}.$$

References:


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